

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2076 Chaitra

Exam.	Regular		
	Level	BE	Full Marks
Programme	All (Except BAR)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III (SH 501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Prove that
$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$
 by using the properties of determinants. [5]

2. Prove that every square complex matrix can uniquely be expressed as a sum of a Hermitian and a skew-Hermitian matrix. [5]

3. Reduce the matrix
$$\begin{bmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{bmatrix}$$
 into normal form and hence find its rank. [5]

4. Find the eigen values and eigen vectors of the matrix
$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$
 and also find its modal matrix. [5]

5. If $\vec{F} = 3x^2yz^2\vec{i} + x^3z^2\vec{j} + 2x^3yz\vec{k}$, show that $\int_C \vec{F} \cdot d\vec{r}$ is independent of the path of integration. Hence evaluate the integral on any path C from (0, 0, 0) to (1, 2, 3). [5]

6. Verify Green's Theorem in plane for $\int_C [(x-y)dx + (x+y)dy]$ where c is the boundary of the region enclosed by $y^2 = x$ and $x^2 = y$. [5]

7. Evaluate $\iint_S \vec{F} \cdot \vec{n} ds$ where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0, z = 3$. [5]

8. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where c is the rectangle bounded by the lines $x = \pm a, y = 0, y = n$ and $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$. [5]

9. State the condition for existence of Laplace transform. Obtain the Laplace transform of:

a) $\cos^3 2t$ (b) $\frac{\cos at - \cos bt}{t}$ [1+1.5+2.5]

10. Find the inverse Laplace transform of:

a) $\frac{s+3}{(s^2+6s+13)^2}$

b) $\frac{e^{-2s}}{(s+1)(s^2+2s+2)}$

[2+3]

11. Solve the differential equation $y''+2y'-3y = \sin t$ under the conditions $y(0) = y'(0) = 0$ by using Laplace transform. [5]

12. Obtain the Fourier series to represent the function $f(x) = e^x$ for $-\pi \leq x \leq \pi$. [5]

13. Obtain the half range cosine series for the function $f(x) = x \sin x$ in the interval $(0, \pi)$. [5]

14. Use Simplex method to solve following LPP:

Maximize, $P = 30x_1 + x_2$

Subject to : $2x_1 + x_2 \leq 10$

$x_1 + 3x_2 \leq 10$

$x_1, x_2 \geq 0$

[7]

15. Use Big M method to solve following LPP:

16. Minimize, $Z = 4x_1 + 2x_2$

Subject to : $3x_1 + x_2 \geq 27$

$-x_1 - x_2 \leq -21$

$x_1 + 2x_2 \geq 30$

$x_1, x_2 \geq 0$

[8]

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	All except BAR	Pass Marks	32
Year / Part	II / 1	Time	3 hrs.

Subject: - Engineering Math III (SH 501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. If $\begin{vmatrix} a & a^2 & a^3 - 1 \\ b & b^2 & b^3 - 1 \\ c & c^2 & c^3 - 1 \end{vmatrix} = 0$, where $a \neq b \neq c$ show that $abc=1$. [5]
2. If A is a square matrix of order n, prove that $A(\text{adj. } A) = (\text{adj. } A)A = |A|I_n$, where I_n is a unit matrix having same order as A. [5]
3. Test the consistency of the system by matrix rank method and solve completely if found consistent: $x+2y-z=3$, $2x+3y+z=10$, $3x-y-7z=1$ [5]
4. State Cayley-Hemilton Theorem and verify it for the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ [1+4]
5. A vector field is given by $\vec{F} = \sin y \vec{i} + x(1 + \cos y) \vec{j}$. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ over the circular path c given by $x^2 + y^2 = a^2$, $z=0$. [5]
6. State and prove Green's Theorem in plane. [1+4]
7. Evaluate $\iint_S \vec{F} \cdot \vec{n} \, ds$ for $\vec{F} = yz \vec{i} + zx \vec{j} + xy \vec{k}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. [5]
8. State Stoke's theorem. Evaluate $\oint_C (xydx + xy^2dy)$ by Stoke's theorem taking c to be a square in the xy-plane with vertices (1,0), (-1,0), (0,1) and (0,-1). [1+4]
9. Find the Laplace transform of : [2+3]
 - i) $te^{-t} \sin t$
 - ii) $\frac{\cos 2t - \cos 3t}{t}$
10. Find the inverse Laplace transform of : [2+3]
 - i) $\frac{s+2}{(s+1)^4}$
 - ii) $\cot^{-1}(s+1)$
11. Solve the differential equation $y'' + y = \sin 3t$, $y(0) = y'(0) = 0$ by using Laplace transform. [5]
12. Define Fourier Series for a function $f(x)$. Obtain Fourier series for $f(x) = x^3$; $-\pi \leq x \leq \pi$. [5]
13. Express $f(x) = e^x$ as the half range Fourier Sine series in $0 < x < 1$. [5]
14. Find the maximum and minimum values of the function $z = 50x_1 + 80x_2$ subject to: $x_1 + 2x_2 \leq 32$, $3x_1 + 4x_2 \leq 84$, $x_1, x_2 \geq 0$; by graphical method. [5]
15. Solve the following Linear Programming problem using big M method: [10]

Maximize $P = 2x_1 + x_2$
Subject to : $x_1 + x_2 \leq 10$
 $-x_1 + x_2 \geq 2$
 $x_1, x_2 \geq 0$

Exam.	Back		
Level	BE	Full Marks	80
Programme	All except BAR	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III (SH 501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Prove that:
$$\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix} = 2(ab+bc+ca)^2$$
 [5]

2. Prove that the necessary and sufficient condition for a square matrix A to possess an inverse is that $|A| \neq 0$. [5]

3. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ by reducing it to normal form. [5]

4. State any two properties of eigen values of a matrix. Obtain eigen values and eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ [1+4]

5. Prove that the line integral $\int_A^B \vec{F} \cdot d\vec{r}$ is independent of path joining any two points A and B in the region if and only if $\oint_C \vec{F} \cdot d\vec{r} = 0$ for any simple closed curve C in the region. [5]

6. State Green's Theorem and use it to find the area of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$. [1+4]

7. Use Gauss' divergence theorem to evaluate $\iiint_S \vec{F} \cdot \vec{n} ds$ where

$\vec{F} = (2xy + z)\vec{i} + y^2\vec{j} - (x + 3y)\vec{k}$ and S is the surface bounded by the plane $2x + 3y + z = 6$, $x=0, y=0, z=0$. [5]

8. Verify Stoke's Theorem for the vector field $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = 1$ bounded by its projection on xy-plane. [5]

9. Find the Laplace transform of: [2+3]

i) $t^2 \cos at$

ii) $\frac{1 - \cosh(at)}{t}$

10. Find the inverse Laplace transform of :

[2+3]

i) $\frac{e^{-\pi s}(s+1)}{s^2+2s+2}$

ii) $\tan^{-1} \frac{2}{s}$

11. Solve the differential equation $y''+3y'+2y=e^t$, $y(0)=y'(0)=0$ by applying Laplace transform. [5]

12. Find the Fourier Series of the function $f(x)=|\sin x|$ for $-\pi \leq x \leq \pi$. [5]

13. If $f(x) = lx-x^2$ in $(0,1)$, show that the half range sine series for $f(x)$ is

$$\frac{8l^2}{\pi^3} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} \sin(2n+1) \frac{\pi x}{1}. \quad [5]$$

14. Find the maximum and minimum values of the function $z=20x+10y$ subject to: $x+2y \leq 40$, $3x+y \geq 30$, $4x+3y \geq 60$, $x, y \geq 0$ by graphical method. [5]

15. Solve the following linear programming problem using big M method:

Maximize $P = 2x_1 + 5x_2$

subject to : $x_1 + 2x_2 \leq 18$

$2x_1 + x_2 \geq 21$

$x_1, x_2 \geq 0$.

[10]

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III (SH501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. If $\begin{vmatrix} a & a^2 & a^3 - 1 \\ b & b^2 & b^3 - 1 \\ c & c^2 & c^3 - 1 \end{vmatrix} = 0$ where $a \neq b \neq c$; apply properties of determinant to show $abc = 1$. [5]

2. If A be an $n \times n$ matrix, prove that

$$\text{Adj}(A) \cdot A = A \cdot (\text{Adj}A) = |A| I \text{ where } I \text{ is an } n \times n \text{ unit matrix.} \quad [5]$$

3. Find the rank of the following matrix by reducing it into normal form:

$$\begin{pmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \\ 1 & 2 & 4 \end{pmatrix} \quad [5]$$

4. Find the modal matrix for the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ -2 & 1 & 3 \\ 2 & 1 & -1 \end{pmatrix} \quad [5]$$

5. State and prove Green's theorem in plane. [5]

6. Find the total work done in moving the particle in a force field given by $\vec{F} = \sin y \vec{i} + x(1 + \cos y) \vec{j}$ over the circular path $x^2 + y^2 = a^2, z = 0$. [5]

7. Evaluate $\iint_S \vec{F} \cdot d\vec{s}$ where $\vec{F} = x \vec{i} - y \vec{j} + z \vec{k}$ and s is the surface of the cylinder $x^2 + y^2 = a^2, 0 < z < b$. [5]

8. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2) \vec{i} - 2xy \vec{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. [5]

9. Obtain Fourier series for $f(x) = x^3$ in the interval $-\pi \leq x \leq \pi$. [5]

10. Express $f(x) = e^x$ as a half range Fourier Cosine Series in $0 < x < 1$. [5]

11. State existence theorem for Laplace Transform. Obtain the Laplace transform of

a) $t e^{-t} \sin t$

b) $\frac{e^{-at} - e^{-bt}}{t}$

1+2+2]

12. Find the inverse Laplace transform of:

a) $\frac{1}{s^2 - 5s + 6}$

b) $\tan^{-1} \frac{2}{s}$

[2+5.+2.5]

13. By using Laplace transform, solve the initial value problem:

$$y'' + 2y = r(t), y(0) = y'(0) = 0$$

$$\text{Where } r(t) = 1, 0 < t < 1$$

$$= 0, \text{ otherwise}$$

[5]

14. Graphically maximize $Z = 5x_1 + 3x_2$ Subject to constraints

$$x_1 + 2x_2 \leq 50$$

$$2x_1 + x_2 \leq 40.$$

$$x_1, x_2 \geq 0$$

[5]

15. Solve the following Linear Programming Problem by simple method:

$$\text{Maximize : } Z = 4x + 3y$$

$$\text{Subject to : } 2x + 3y \leq 6$$

$$-x + 2y \leq 3$$

$$2y \leq 5$$

$$2x + y \leq 4$$

$$x, y \geq 0.$$

[10]

Exam.	Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III (SH501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Define the determinant as a function and using its properties. Show that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \quad [5]$$

2. If A and B are orthogonal matrices of same order, prove that the product AB is also orthogonal. [5]

3. Test the consistency of the system $x - 2y + 2z = 4$, $3x + y + 4z = 6$ and $x + y + z = 1$ and solve completely if found consistent. [5]

4. For a matrix $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$, find the modal matrix and the corresponding diagonal matrix. [5]

5. Prove that line integral $\int_A^B \vec{F} \cdot d\vec{r}$ is independent of path joining any two points A and B in the region if and only if $\int_C \vec{F} \cdot d\vec{r} = 0$ for any simple closed curve C in the region. [5]

6. Verify Green's theorem in the plane for $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is region bounded by $y = x^2$ and $x = y^2$. [5]

7. Evaluate $\iint_S \vec{F} \cdot \vec{n} \, ds$ where $\vec{F} = 6z\vec{i} - 4\vec{j} + y\vec{k}$ and S is the region of the plane $2x + 3y + 6z = 12$ bounded in the first octant. [5]

8. Evaluate using Gauss divergence theorem, $\iiint_S \vec{F} \cdot \vec{n} \, ds$ where $\vec{F} = x^2y\vec{i} + xy^2\vec{j} + 2xyz\vec{k}$ and S is the surface bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + 2y + z = 2$. [5]

9. Obtain the Fourier Series to represent $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$ and deduce that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots \quad [5]$$

10. Obtain the half range Fourier Sine Series for $f(x) = \pi - x$ in the range $0 < x < \pi$. [5]

11. State the conditions for existence of Laplace transform. Obtain the Laplace transform of:

(i) $e^{2t} \cos^3 2t$ (ii) $\frac{\cos 2t - \cos 3t}{t}$ [1+2+2]

12. Find the inverse Laplace transform of:

(i) $\frac{1}{(S-2)(S^2+1)}$ (ii) $\cot^{-1}(S+1)$

[2.5+2.5]

13. Solve the following initial value problem by using Laplace transform:

$$y'' + 4y' + 3y = e^t, \quad y(0) = 0; y'(0) = 2$$

[5]

14. Graphically maximize $Z = 7x_1 + 10x_2$

Subject to constraints:

$$3x_1 + x_2 \leq 9$$

$$x_1 + 2x_2 \leq 8$$

$$x_1, x_2 \geq 0.$$

[5]

15. Solve the following linear Programming Problem by simple method:

Maximize: $Z = 3x_1 + 5x_2$

Subject to:

$$3x_1 + 2x_2 \leq 18$$

$$x_1 \leq 4, \quad x_2 \leq 6$$

$$x_1, x_2 \geq 0.$$

[10]

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III (SH501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Use properties of determinants to prove: [5]

$$\begin{vmatrix} a^2+1 & ba & ca & da \\ ab & b^2+1 & cb & db \\ ac & bc & c^2+1 & dc \\ ad & bd & cd & d^2+1 \end{vmatrix} = 1+a^2+b^2+c^2+d^2$$

2. Show that every square matrix can be uniquely expressed as the sum of symmetric and Skew-Symmetric matrices. [5]

3. Test the consistency of the system $x+y+z=3$, $x+2y+3z=4$ and $2x+3y+4z=7$ and solve completely if found consistent. [5]

4. State Cayley-Hamilton theorem and verify it for the matrix; $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$ [1+4]

5. Prove that " The line integral $\int_C \vec{F} \cdot d\vec{r}$ of a continuous function \vec{F} defined in a region R is independent of path C joining any two points in R if and only if there exists a single valued scalar function ϕ having first order partial derivatives such that $\vec{F} = \nabla\phi$ ". [5]

6. State Green's theorem and use it to find the area of astroid $x^{2/3} + y^{2/3} = a^{2/3}$ [5]

7. Evaluate $\iint_S \vec{F} \cdot \vec{n} \, ds$, where $\vec{F} = x^2 \vec{i} + y^2 \vec{j} + z^2 \vec{k}$ and 's' is the surface of the plane $x+y+z=1$ between the co-ordinate planes. [5]

8. Apply Gauss' divergence theorem to evaluate $\iiint_S \vec{F} \cdot \vec{n} \, ds$ where

$\vec{F} = (x^3 - yz) \vec{i} - 2x^2y \vec{j} + 2z \vec{k}$ and 's' is the surface the cube bounded by the planes $x=0, x=a, y=0, y=a, z=0, z=a$. [5]

9. Find the Laplace transform of: [2+3]

i) $t \sin^2 3t$

ii) $\frac{\sin 2t}{t}$

10. Find the inverse Laplace transform of: [2+3]

i) $\frac{1}{s^2 - 3s + 2}$

ii) $\frac{1}{s(s+1)^3}$

11. Apply Laplace transform to solve the differential equation: [5]

$$y'' + 2y' + 5y = e^{-t} \sin t, \quad x(0) = 0, x'(0) = 1$$

12. Find a Fourier series to represent $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12} \quad [5]$$

13. Develop $f(x) = \sin\left(\frac{\pi x}{l}\right)$ in half range Cosine Series in the range $0 < x < l$. [5]

14. Graphically maximize, [5]

$$Z = 7x_1 + 10x_2$$

Subject to constraints,

$$3x_1 + x_2 \leq 9$$

$$x_1 + 2x_2 \leq 8$$

$$x_1 \geq 0, x_2 \geq 0$$

15. Solve the following LPP using simplex method. [10]

$$\text{Maximize: } P = 50x_1 + 80x_2$$

$$\text{Subject to: } x_1 + 2x_2 \leq 32$$

$$3x_1 + 4x_2 \leq 84$$

$$x_1 \geq 0, x_2 \geq 0$$

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	ALL (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics II (SH501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt **All** questions.
- ✓ The figures in the margin indicate **Full Marks**.
- ✓ Assume suitable data if necessary.

1. Use properties of determinants to prove:
$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$
 [5]

2. Prove that the necessary and sufficient condition for a square matrix A to possess an inverse is that the matrix A should be non singular. [5]

3. Find the rank of the matrix
$$\begin{pmatrix} 1 & 3 & -2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{pmatrix}$$
 [5]

by reducing it into normal form.

4. Find the eigenvalues and eigenvectors of the matrix
$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$
 [4+1]

Give an example showing importance of eigenvectors.

5. Show that $\vec{F} = (2x+z^2)\vec{i} + z\vec{j} + (y+2xz)\vec{k}$ is irrotational and find its scalar potential. [5]

6. State and prove Green's Theorem in plane. [5]

7. Evaluate $\iint_S \vec{F} \cdot \vec{n} \, ds$, where $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. [5]

8. Evaluate $\int_C xydx + xy^2dy$ by applying Stokes theorem where C is the square in xy-plane with vertices (1,0), (-1,0), (0,1), (0,-1) [5]

9. Find the Laplace transform of: [2+3]

i) $te^{2t} \sin 3t$

ii) $\frac{e^{-t} \sin t}{t}$

10. Find the inverse Laplace transform of :

[2+3]

i) $\frac{s+2}{s^2-4s+13}$

ii) $\log\left(\frac{s+a}{s-a}\right)$

11. Solve the following initial value problem using Laplace transform:

[5]

$$x''+4x'+4x=6e^{-t}, \quad x(0)=-2, \quad x'(0)=-8$$

12. Find the Fourier series representation of $f(x)=|x|$ in $[-\pi, \pi]$

[5]

13. Obtain the half range Fourier Sine Series for the function $f(x)=x^2$ in the interval $(0, 3)$.

[5]

14. Apply Graphical method to maximize,

[5]

$$Z=5x_1+3x_2$$

Subject to the constraints:

$$x_1+2x_2 \leq 50$$

$$2x_1+x_2 \leq 40$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

15. Solve the following Linear Programming Problem by Simplex method:

[10]

$$\text{Maximize: } Z=15x_1+10x_2$$

$$\text{Subject to: } x_1+3x_2 \leq 10$$

$$2x_1+x_2 \leq 10$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III (SH501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Prove that
$$\begin{vmatrix} (a+b)^2 & ca & bc \\ ca & (b+c)^2 & ab \\ bc & ab & (c+a)^2 \end{vmatrix} = 2abc(a+b+c)^3$$
 [5]

2. If A and B are two non singular matrices, then prove that $(AB)^{-1} = B^{-1}A^{-1}$ [5]

3. Find the rank of the matrix: [5]

$$\begin{pmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

4. Find the eigen values and eigen vectors of the matrix. [5]

$$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

5. Prove that the line integral $\int_A^B \vec{F} \cdot d\vec{r}$ is independent of path joining any two points A and B in the region R, if and only if, $\int_C \vec{F} \cdot d\vec{r} = 0$ for any simple closed path C in R. [5]

6. Evaluate $\iint_S \vec{F} \cdot \vec{n} \, ds$ where $\vec{F} = yz \vec{i} + zx \vec{j} + xy \vec{k}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. [5]

OR

Apply Stoke's theorem to evaluate $\int_C (x+y)dx + (2x-z)dy + (y+z)dz$ where C is the boundary of the triangle with vertices (2,0,0), (0,3,0) and (0,0,6). [5]

7. State Green's theorem in plane and hence apply it to compute the area of the curve $x^{2/3} + y^{2/3} = a^{2/3}$. [5]

8. Apply Gauss divergence theorem to evaluate $\iiint_S \vec{F} \cdot \vec{n} \, ds$ where $\vec{F} = x^2 \vec{i} + z \vec{j} + yz \vec{k}$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. [5]

9. Find the Laplace transform of the following: [2.5×2]

a) $\frac{\cos 2t - \cos 3t}{t}$

b) $\sin^3 2t$

10. Find the inverse Laplace transform of the following: [2+3]

a) $\frac{1}{s^2 - 5s + 6}$

b) $\frac{s+2}{(s^2 + 4s + 5)^2}$

11. Solve the initial value problem by using Laplace transform: [5]

$$x'' + 2x' + 5x = e^{-t} \sin t; \quad x(0) = 0, \quad x'(0) = 1$$

12. Obtain Fourier Series for the function $f(x) = x - x^2$ from $-\pi$ to π and hence show that: [5]

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

13. Obtain the half range sine series for the function $f(x) = x^2$ in the interval (0,3). [5]

14. Graphically maximize and minimize [5]

$$Z = 5x_1 + 3x_2 \text{ Subjected to constraints}$$

$$3x_1 + 5x_2 \leq 15$$

$$5x_1 + 2x_2 \leq 10, x_1, x_2 \geq 0$$

15. Use simplex method to solve the Linear Programming problem: [10]

$$\text{Maximize } Z = 15x_1 + 10x_2$$

$$\text{Subject to } 2x_1 + 2x_2 \leq 10$$

$$x_1 + 3x_2 \leq 10$$

$$\text{and } x_1, x_2 \geq 0$$

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III (SH501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Using the properties, evaluate the determinant: [5]

$$\begin{vmatrix} 1 & a & a^2 & a^3 + bcd \\ 1 & b & b^2 & b^3 + cda \\ 1 & c & c^2 & c^3 + abd \\ 1 & d & d^2 & d^3 + abc \end{vmatrix}$$

2. Prove that every square matrix can uniquely be expressed as the sum of a symmetric and a skew symmetric matrix. [5]

3. Test the consistency of the system: [5]

$$x - 6y - z = 10, \quad 2x - 2y + 3z = 10, \quad 3x - 8y + 2z = 20$$

And solve completely, if found consistent.

4. Find the eigen values and eigenvectors of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$. [5]

5. Using the line integral, compute the workdone by the force [5]

$$\vec{F} = (2x - y + 2z)\vec{i} + (x + y - z)\vec{j} + (3x - 2y - 5z)\vec{k}$$

when it moves once around a circle $x^2 + y^2 = 4; z = 0$

6. State and prove Green's Theorem in plane. [5]

7. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. [5]

8. Evaluate $\iint_S \vec{F} \cdot \vec{n} \, ds$ where $\vec{F} = (2xy + z)\vec{i} + y^2\vec{j} - (x + 3y)\vec{k}$ by Gauss divergence theorem; where S is surface of the plane $2x + 2y + z = 6$ in the first octant bounding the volume V. [5]

9. Find the Laplace transform of the following: [2.5×2]

- a) $te^{-2t} \cos t$
- b) $\text{Sinhat} \cdot \cos t$

10. Find the inverse Laplace transform of :

[2.5×2]

a) $\frac{1}{S(S+1)}$

b) $\frac{S^2}{(S^2 + b^2)^2}$

11. Solve the differential equation $y''+2y'+5y=e^{-t}\sin t, y(0)=0, y'(0)=1$, by using Laplace transform. [5]

12. Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \leq x \leq \pi$. [5]

13. Obtain half range sine series for the function $f(x) = x - x^2$ for $0 < x < 1$. [5]

14. Graphically maximize and minimize [5]

$z = 9x + 40y$ subjected to the constraints

$$y - x \geq 1, y - x \leq 3, 2 \leq x \leq 5$$

15. Solve the following Linear Programming Problem by Simplex method: [10]

Maximize, $P = 20x_2 - 5x_1$

Subjected to, $10x_2 - 2x_1 \leq 5$

$$2x_1 + 5x_2 \leq 10 \text{ and } x_1, x_2 \geq 0$$

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Mathematics III (SH501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Show that:
$$\begin{vmatrix} (b+c)^2 & b^2 & c^2 \\ a^2 & (c+a)^2 & c^2 \\ a^2 & b^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$
 [5]

2. Prove that every square matrix can be uniquely written as a sum of Hermitian and Skew-Hermitian matrices. [5]

3. Find the rank of the matrix by changing it into normal form:
$$\begin{pmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \\ 1 & 2 & 4 \end{pmatrix}$$
 [5]

4. Find the eigen value and eigen vector of the matrix:
$$\begin{pmatrix} 2 & 1 & 1 \\ -2 & 1 & 3 \\ 2 & 1 & -1 \end{pmatrix}$$
 [5]

5. Using Green's theorem, evaluate $\int_C (y^3 dx - x^3 dy)$ where C is the boundary of the circle $x^2 + y^2 = 4$. [5]

6. Show that $\vec{F}(x, y, z) = y^3 \vec{i} + (3xy^2 + e^{2z}) \vec{j} + 2ye^{2z} \vec{k}$ is conservative vector field and find its scalar potential function. [5]

7. Find the surface integral $\iint_S \vec{F} \cdot \hat{n} \, ds$ where $\vec{F} = x \vec{i} + y \vec{j} + z \vec{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$. [5]

8. Verify Stoke's theorem for $\vec{F}(x, y, z) = (2x - y) \vec{i} - yz^2 \vec{j} - y^2z \vec{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 4$ and C is its boundary. [5]

OR

Evaluate using Gauss divergence theorem,

$$\iint_S \vec{F} \cdot \hat{n} \, ds$$
 where $\vec{F}(x, y, z) = x^2y \vec{i} + xy^2 \vec{j} + 2xyz \vec{k}$ and S is the surface bounded by the planes $x = 0, y = 0, z = 0$ and $x + 2y + z = 2$

9. Find the Laplace transform of (i) $\sin 2t \cosh 4t$ (ii) $te^{2t} \sin 4t$. [5]

10. Using the Convolution theorem, find the inverse Laplace transform of $\frac{3s}{(s^2 + 4)(s^2 + 1)}$ [5]

11. Solve the following initial value problem using Laplace transform: [5]

$$y'' + 4y' + 3y = e^t, y(0) = 0, y'(0) = 2$$

12. Obtain the half range Fourier sine series of $f(x) = \pi - x$ in the range $0 < x < \pi$. [5]

13. Obtain the Fourier series of $f(x) = e^{3x}$ in $0 < x < 2\pi$. [5]

14. Graphically maximum $Z = 5x_1 + 3x_2$ subject to constraints [5]

$$x_1 + 2x_2 \leq 50, 2x_1 + x_2 \leq 40 \text{ and } x_1 \geq 0, x_2 \geq 0$$

15. Solve the following linear programming problem by simplex method constructing the duality: [10]

$$\text{Minimize: } P = 21x_1 + 50x_2$$

$$\text{Subject to } 3x_1 + 7x_2 \geq 17$$

$$2x_1 + 5x_2 \geq 12$$

$$x_1, x_2 \geq 0$$

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Mathematics III (SH501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Using the properties of determinant prove [5]

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

2. Prove that $(AB)^T = B^T A^T$ where A is the matrix of size $m \times p$ and B is the matrix of size $p \times n$ [5]

3. Find the rank of the following matrix by reducing normal form. [5]

$$\begin{bmatrix} 1 & 3 & -2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{bmatrix}$$

4. Find the eigen values and eigen vectors of the following matrix. [5]

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

5. Prove that the line intergral $\int_A^B \vec{F} \cdot d\vec{r}$ is independent of the path joining any two points A and B in a region if $\int_C \vec{F} \cdot d\vec{r} = 0$ for any simple closed curve C in the region. [5]

6. Evaluate $\iint_S \vec{F} \cdot \hat{n} \, ds$ where $\vec{F} = x^2 \vec{i} + y^2 \vec{j} + z^2 \vec{k}$ and S is the finite plane $x + y + z = 1$ between the coordinate planes. [5]

OR

Evaluate $\iint_S \vec{F} \cdot \hat{n} \, ds$ for $\vec{F} = yz \vec{i} + zx \vec{j} + xy \vec{k}$ where S is the surface of sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

7. Evaluate, $\iint_S \vec{F} \cdot \hat{n} \, ds$ for $\vec{F} = x \vec{i} - y \vec{j} + (z^2 - 1) \vec{k}$ where S is the surface bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 1$ [5]

8. Verify the stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the upper part of the sphere $x^2 + y^2 + z^2 = a^2$ C is its boundary. [5]

9. Find the Laplace transform of (a) $t^2 \sin zt$ and (b) $\frac{1-e^t}{t}$ [2.5×2]

10. Find the inverse Laplace transform of (a) $\frac{2s+3}{s^2+5s-6}$ (b) $\frac{s^3}{s^4-a^4}$ [2.5×2]

11. Solve the following differential equation by using Laplace transform [5]

$$y'' + y' - 2y = x, y(0) = 1, y'(0) = 0$$

12. Obtain the Fourier series for $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence prove that

$$\sum \frac{1}{x^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \dots = \frac{\pi^2}{6} \quad [5]$$

13. Obtain half range sine series for $f(x) = \pi x - x^2$ in $(0, \pi)$ [5]

14. Graphically minimize $z = 4x_1 + 3x_2 + x_3$ [5]

$$\text{Subject to } x_1 + 2x_2 + 4x_3 \geq 12$$

$$3x_1 + 2x_2 + x_3 \geq 8 \text{ and } x_1, x_2, x_3 \geq 0$$

15. Minimize $z = 8x_1 + 9x_2$ [10]

$$\text{Subject to } x_1 + 3x_2 \geq 4$$

$$2x_1 + x_2 \geq 5 \text{ with } x_1, x_2 \geq 0$$

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III (SH501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Prove that:
$$\begin{vmatrix} a & b & b & b \\ a & b & a & a \\ a & a & b & a \\ b & b & b & a \end{vmatrix} = -(b-a)^4$$
 [5]

2. Prove that every matrix A can uniquely be expressed as a sum of a symmetric and a skew symmetric matrix. [5]

3. Test the consistency of the system $x+y+z = 3$, $x+2y+3z = 4$ and $2x+3y+4z = 7$ and solve if consistent. [5]

4. Verify Cayley-Hamilton theorem for matrix A and find the inverse of
$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$
 [5]

5. State and prove Green's theorem in the plane. [5]

OR

Verify Stroke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle in the xy-plane bounded by $x = 0$, $x = a$, $y = 0$, $y = b$

6. Find the work done in moving particle once round the circle $x^2 + y^2 = 9$, $z = 0$ under the force field \vec{F} given by $\vec{F} = (2x - y + z)\vec{i} + (x + y - z^2)\vec{j} + (3x - 2y + 4z)\vec{k}$ [5]

7. Evaluate $\iint_s \vec{F} \cdot \vec{n} \, ds$ where $\vec{F} = xy\vec{i} - x^2\vec{j} + (x+z)\vec{k}$, s is the portion of the plane $2x+2y+z = 6$ included in the first octant. [5]

8. Show that $\iint_s \left[(x^3 - yz)\vec{i} - 2x^2y\vec{j} + 2z\vec{k} \right] \cdot \vec{n} \, ds = \frac{a^5}{3}$ where s is the surface of the cube bounded by the planes $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$, $z = a$ [5]

9. Find the Laplace transform of (i) $f(t) = \frac{1 - \cos t}{t}$ (ii) $f(t) = te^{-t} \sin t$ [5]

10. Find the inverse Laplace transform of (i) $\frac{(s+2)^3}{s^4}$ (ii) $\frac{1}{s^2(s^2+a^2)}$ [5]

11. Using Laplace Transform to solve: $y''+4y = \sin t$; $y(0) = 0 = y'(0)$ [5]

12. Find a fourier series to represent $f(x) = x - x^2$ from $x = -\Pi$ to $x = \Pi$ [5]

13. Find a fourier series to represent $f(x) = 2x - x^2$ in the range $(0,3)$ [5]

OR

Express $f(x) = x$ as a half range sine series in $0 < x < \Pi$

14. Use simplex method to, Maximize $p = 15x_1 + 10x_2$ [7]

Subject to $2x_1 + x_2 \leq 10$

$x_1 + 3x_2 \leq 10, \quad x_1, x_2 \geq 0$

15. Find the dual of following Linear programming problem and solve by simplex method [8]

Minimize $C = 16x_1 + 45x_2$

Subject to $2x_1 + 5x_2 \geq 50$

$x_1 + 3x_2 \geq 27, \quad x_1, x_2 \geq 0$

OR

Use Big M-method to solve the following linear programming problem.

Maximize $p = 2x_1 + x_2$

Subject to $x_1 + x_2 \leq 10$

$-x_1 + x_2 \geq 2, \quad x_1, x_2 \geq 0$

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III (SH501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Find the value of the determinant $\begin{vmatrix} a^2 & a^2 - (b-c)^2 & bc \\ b^2 & b^2 - (c-a)^2 & ca \\ c^2 & c^2 - (a-b)^2 & ab \end{vmatrix}$ [5]

2. Show that the matrix $B^0 AB$ is Hermitian or skew-Hermitian according as A is Hermitian and skew-Hermitian. [5]

3. Find the rank of the matrix $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ reducing this into the triangular form. [5]

4. Obtain the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and verify that it is satisfied by A. [5]

5. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (x-y)\vec{i} + (x+y)\vec{j}$ along the closed curve C bounded by $y^2 = x$ and $x^2 = y$ [5]

6. Find the value of the normal surface integral $\iint_S \vec{F} \cdot \vec{n} \, ds$ for $\vec{F} = x\vec{i} - y\vec{j} + (z^2 - 1)\vec{k}$, where S is the surface bounded by the cylinder $x^2 + y^2 = 4$ between the planes $Z = 0$ and $Z = 1$. [5]

7. Using Green's theorem, find the area of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ [5]

8. Verify stoke's theorem for $\vec{F} = 2y\vec{i} + 3x\vec{j} - z^2\vec{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 9$ and C is its boundary. [5]

OR

Evaluate the volume intergral $\iiint_V \vec{F} \, dv$, where V is the region bounded by the surface $x=0, y=0, y=6, z=x^2, z=4$ and $\vec{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$

9. Find the Laplace transforms of the following functions [2.5×2]
- a) $t e^{-4t} \sin 3t$
 - b) $\frac{\cos at - \cos bt}{t}$

10. State and prove the second shifting theorem of the Laplace transform. [5]

11. Solve the following differential equation using Laplace transform. [5]

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x \text{ given } y(0) = 1, y'(0) = 0$$

12. Obtain the Fourier series for $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence show that

$$\sum \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6} \quad [5]$$

13. Express $f(x) = x$ as a half-range sine series in $0 < x < 2$ [5]

14. Maximize $Z = 4x_1 + 5x_2$ subject to constraints [5]

$$2x_1 + 5x_2 \leq 25$$

$$6x_1 + 5x_2 \leq 45$$

$$x_1 \geq 0 \text{ and } x_2 \geq 0$$

graphically

15. Solve the following linear programming problem using the simplex method. [10]

$$\text{Maximize } P = 50x_1 + 80x_2$$

$$\text{Subject to } x_1 + 2x_2 \leq 32$$

$$3x_1 + 4x_2 \leq 84$$

$$x_1, x_2 \geq 0$$

02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division

2068 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BCE, BEL, BEX, BCT, BME, BIE, B. AGRI.	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III (SH 501)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Prove that:
$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2. \quad [5]$$

2. Define Hermitian and Skew Hermitian matrix. Show that every square matrix can be uniquely expressed as the sum of a Hermitian and a skew Hermitian. [5]

3. For what value of λ the equation $x + y + z = 1$, $x + 4y + 10z = \lambda^2$ and $x + 2y + 4z = \lambda$ have a solution? Solve them completely in each case. [5]

4. Find the eigen values and eigen vectors of $A = \begin{vmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{vmatrix}$. [5]

5. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, Where $C: x^2 = y$ and $y^2 = x$ and $\vec{F} = (x-y)\vec{i} + (x+y)\vec{j}$. [5]

6. State and prove Green theorem in a plane. [5]

7. Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + 3y\vec{j} + yz\vec{k}$. Taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. [5]

8. Find the Laplace transform of the given function (i) $t^2 \sin t$ (ii) $\cos at \sin hat$. [5]

9. Evaluate $\iiint_S \vec{F} \cdot \hat{n} ds$ where $\vec{F} = 3x\vec{i} + y\vec{j} - yz\vec{k}$ and s is the surface of the cylinder $x^2 + y^2 = 9$ included in the first octant between the plane $z = 0, z = 4$. [5]

10. Find the inverse Laplace transform: (a) $\frac{1}{(S-2)(S+4)}$ (b) $\log\left(\frac{s^2 + a^2}{s^2}\right)$ [5]

11. Solve the equation using Laplace transform $y'' + 4y' + 3y = t, t > 0, y(0) = 0, y'(0) = 1$. [5]

12. Obtain a Fourier series to represent the function $f(x) = |x|$ for $-\pi \leq x \leq \pi$ and hence deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ [5]

13. Obtain the half Range Sine Series $f(x) = ex$ in $0 < x < 1$. [5]

OR

Obtain the Fourier series for $f(x) = x - x^2$ where $-1 < x < 1$ as a Fourier series of period 2.

14. Solve the following by using the simplex method: [7.5]

Maximize $P = 15x_1 + 10x_2$,

Subject to

$2x_1 + x_2 \leq 10$,

$x_1 + 3x_2 \leq 10$,

$x_1, x_2 \geq 0$.

15. Solve by using the dual method: [7.5]

Minimize $C = 21x_1 + 50x_2$,

Subject to $2x_1 + 5x_2 \leq 12$,

$3x_1 + 7x_2 \leq 17$,

$x_1, x_2 \geq 0$.

OR

Solve the following LPP by using the big M-method:

Maximize $P = 2x_1 + x_2$,

Subject to

$x_1 + x_2 \leq 10$,

$-x_1 + x_2 \geq 2$,

$x_1, x_2 \geq 0$.
