TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2076 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except BAR)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$ by using the properties of determinants. [5]

2. Prove that every square complex matrix can uniquely be expressed as a sum of a Hermitian and a skew-Hermitian matrix. [5]

3. Reduce the matrix $\begin{bmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{bmatrix}$ into normal form and hence find its rank. [5]

4. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$ and also find its modal matrix. [5]

5. If $\vec{F} = 3x^2yz^2\,\vec{i} + x^3z^2\,\vec{j} + 2x^3yz\,\vec{k}$, show that $\int_c \vec{F}.d\vec{r}$ is independent of the path of integration. Hence evaluate the integral on any path C from (0, 0, 0) to (1, 2, 3). [5]

6. Verify Green's Theorem in plane for $\int_c [(x-y)\,dx + (x+y)\,dy]$ where c is the boundary of the region enclosed by $y^2 = x$ and $x^2 = y$. [5]

7. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ where $\vec{F} = 4x\,\vec{i} - 2y^2\,\vec{j} + z^2\,\vec{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0, z = 3$. [5]

8. Evaluate $\int_c \vec{F}.d\vec{r}$, where c is the rectangle bounded by the lines $x = \pm a, y = 0, y = n$ and $\vec{F} = (x^2+y^2)\,\vec{i} - 2xy\,\vec{j}$. [5]

9. State the condition for existence of Laplace transform. Obtain the Laplace transform of:

   a) $\cos^3 2t$ (b) $\frac{\cos at - \cos bt}{t}$ [1+1.5+2.5]

10. Find the inverse Laplace transform of:

a) $\frac{s+3}{(s^2+6s+13)^2}$ b) $\frac{e^{-2s}}{(s+1)(s^2+2s+2)}$ [2+3]

11. Solve the differential equation $y''+2y'-3y=\sin t$ under the conditions $y(0)=y'(0)=0$ by using Laplace transform. [5]

12. Obtain the Fourier series to represent the function $f(x)=e^x$ for $-\pi \le x \le \pi$. [5]

13. Obtain the half range cosine series for the function $f(x)=x\sin x$ in the interval $(0, \pi)$. [5]

14. Use Simplex method to solve following LPP:

Maximize, $P = 30x_1 + x_2$

Subject to : $2x_1 + x_2 \le 10$

$x_1 + 3x_2 \le 10$

$x_1, x_2 \ge 0$ [7]

15. Use Big M method to solve following LPP:

16. Minimize, $Z = 4x_1 + 2x_2$

Subject to : $3x_1 + x_2 \ge 27$

$-x_1 - x_2 \le -21$

$x_1 + 2x_2 \ge 30$

$x_1, x_2 \ge 0$ [8]

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TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2075 Chaitra

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	All except BAR	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Math III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. If $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$, where $a\neq b\neq c$ show that abc=1. [5]

2. If A is a square matrix of order n, prove that A(adj. A) = (adj. A)A = $|A|I_n$, where I_n is a unit matrix having same order as A. [5]

3. Test the consistency of the system by matrix rank method and solve completely if found consistent: x+2y-z=3, 2x+3y+z=10, 3x-y-7z=1 [5]

4. State Cayley-Hemilton Thorem and verify it for the matrix A=$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ [1+4]

5. A vector field is given by $\vec{F} = \sin y\vec{i} + x(1+\cos y)\vec{j}$. Evaluate the line integral $\int_c \vec{F}.d\vec{r}$ over the circular path c given by $x^2+y^2=a^2$, z=0. [5]

6. State and prove Green's Theorem in plane. [1+4]

7. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ for $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ where S is the surface of the sphere $x^2+y^2+z^2=1$ in the first octant. [5]

8. State Stoke's theorem. Evaluate $\oint_c (xydx + xy^2dy)$ by Stoke's theorem taking c to be a square in the xy-plane with vertices (1,0),(-1,0),(0,1) and (0,-1). [1+4]

9. Find the Laplace transform of: [2+3]
   i) $te^{-t}\sin t$
   ii) $\dfrac{\cos 2t - \cos 3t}{t}$

10. Find the inverse Laplace transform of: [2+3]
    i) $\dfrac{s+2}{(s+1)^4}$
    ii) $\cot^{-1}(s+1)$

11. Solve the differential equation y''+y=sin3t, y(0)=y'(0)=0 by using Laplace transform. [5]

12. Define Fourier Series for a function f(x). Obtain Fourier series for $f(x)=x^3$; $-\pi \leq x \leq \pi$. [5]

13. Express $f(x)=e^x$ as the half range Fourier Sine series in 0<x<1. [5]

14. Find the maximum and minimum values of the function $z = 50x_1 + 80x_2$ subject to: $x_1 + 2x_2 \leq 32$, $3x_1 + 4x_2 \leq 84$, $x_1x_2 \geq 0$; by graphical method. [5]

15. Solve the following Linear Programming problem using big M method: [10]
    Maximize $P= 2x_1+x_2$
    Subject to : $x_1+x_2 \leq 10$
    $-x_1+x_2 \geq 2$
    $x_1,x_2 \geq 0$

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TRIBHUVAN UNIVERSITY
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Examination Control Division
2076 Ashwin

Exam.	Back		
Level	BE	Full Marks	80
Programme	All except BAR	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Prove that: $\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix} = 2(ab+bc+ca)^2$ [5]

2. Prove that the necessary and sufficient condition for a square matrix A to possess an inverse is that $|A| \neq 0$. [5]

3. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ by reducing it to normal form. [5]

4. State any two properties of eigen values of a matrix. Obtain eigen values and eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ [1+4]

5. Prove that the line integral $\int_A^B \vec{F}.d\vec{r}$ is independent of path joining any two points A and B in the region if and only if $\int_C \vec{F}.d\vec{r} = 0$ for any simple closed curve C in the region. [5]

6. State Green's Theorem and use it to find the area of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$. [1+4]

7. Use Gauss' divergence theorem to evaluate $\iint_S \vec{F}.\vec{n}ds$ where $\vec{F} = (2xy+z)\vec{i} + y^2\vec{j} - (x+3y)\vec{k}$ and S is the surface bounded by the plane 2x+3y+z=6, x=0, y=0, z=0. [5]

8. Verify Stoke's Theorem for the vector field $\vec{F} = (2x-y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ over the upper half of the sphere $x^2+y^2+z^2=1$ bounded by its projection on xy-plane. [5]

9. Find the Laplace transform of: [2+3]

    i) $t^2\cos at$

    ii) $\frac{1-\cosh(at)}{t}$

10. Find the inverse Laplace transform of: [2+3]

i) $\frac{e^{-\pi s}(s+1)}{s^2+2s+2}$

ii) $\tan^{-1}\frac{2}{s}$

11. Solve the differential equation $y''+3y'+2y=e^{-t}$, $y(0)=y'(0)=0$ by applying Laplace transform. [5]

12. Find the Fourier Series of the function $f(x)=|\sin x|$ for $-\pi \leq x \leq \pi$. [5]

13. If $f(x) = lx-x^2$ in $(0,l)$, show that the half range sine series for $f(x)$ is $\frac{8l^2}{\pi^3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^3}\sin(2n+1)\frac{\pi x}{l}$. [5]

14. Find the maximum and minimum values of the function $z=20x+10y$ subject to: $x+2y\leq40$, $3x+y\geq30$, $4x+3y \geq 60$, $x,y\geq0$ by graphical method. [5]

15. Solve the following linear programming problem using big M method:

Maximize $P=2x_1+5x_2$

subject to : $x_1+2x_2 \leq 18$

$2x_1+x_2 \geq 21$

$x_1,x_2 \geq 0$. [10]

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01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2074 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. If $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$ where $a \neq b \neq c$; apply properties of determinant to show $abc = 1$. [5]

2. If A be an $n \times n$ matrix, prove that

   $Adj(A).A = A.(AdjA) = |A| I$ where I is an $n \times n$ unit matrix. [5]

3. Find the rank of the following matrix by reducing it into normal form:

   $$\begin{pmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \\ 1 & 2 & 4 \end{pmatrix}$$ [5]

4. Find the modal matrix for the matrix

   $$A = \begin{pmatrix} 2 & 1 & 1 \\ -2 & 1 & 3 \\ 2 & 1 & -1 \end{pmatrix}$$ [5]

5. State and prove Green's theorem in plane. [5]

6. Find the total work done in moving the particle in a force field given by $\vec{F} = \sin y\, \vec{i} + x(1+\cos y)\, \vec{j}$ over the circular path $x^2 + y^2 = a^2$, $z = 0$. [5]

7. Evaluate $\iint_S \vec{F}.d\vec{s}$ where $\vec{F} = x\vec{i} - y\vec{j} + z\vec{k}$ and s is the surface of the cylinder $x^2 + y^2 = a^2, 0 < z < b$. [5]

8. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. [5]

9. Obtain Fourier series for $f(x) = x^3$ in the interval $-\pi \leq x \leq \pi$. [5]

10. Express $f(x) = e^x$ as a half range Fourier Cosine Series in $0 < x < 1$. [5]

11. State existence theorem for Laplace Transform. Obtain the Laplace transform of

    a) $te^{-t} \sin t$ b) $\frac{e^{-at} - e^{-bt}}{t}$ 1+2+2]

12. Find the inverse Laplace transform of:

a) $\frac{1}{s^2-5s+6}$ b) $\tan^{-1}\frac{2}{s}$ [2+5.+2.5]

13. By using Laplace transform, solve the initial value problem:

$y'' + 2y = r(t)$, $y(0) = y'(0) = 0$
Where $r(t) = 1$, $0 < t < 1$
$= 0$, otherwise [5]

14. Graphically maximize $Z = 5x_1 + 3x_2$ Subject to constraints

$x_1 + 2x_2 \leq 50$
$2x_1 + x_2 \leq 40$.
$x_1, x_2 \geq 0$ [5]

15. Solve the following Linear Programming Problem by simple method:

Maximize : $Z = 4x + 3y$

Subject to : $2x + 3y \leq 6$
$-x + 2y \leq 3$
$2y \leq 5$
$2x + y \leq 4$
$x, y \geq 0$. [10]

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2075 Ashwin

Exam.	Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Define the determinant as a function and using its properties. Show that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$ [5]

2. If A and B are orthogonal matrices of same order, prove that the product AB is also orthogonal. [5]

3. Test the consistency of the system $x-2y+2z=4$, $3x+y+4z=6$ and $x+y+z=1$ and solve completely if found consistent. [5]

4. For a matrix $A=\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$, find the modal matrix and the corresponding diagonal matrix. [5]

5. Prove that line integral $\int_A^B \vec{F}.d\vec{r}$ is independent of path joining any two points A and B in the region if and only if $\int_C \vec{F}.d\vec{r}=0$ for any simple closed curve C in the region. [5]

6. Verify Green's theorem in the plane for $\int_C [(3x^2-8y^2)dx+(4y-6xy)dy]$ where C is region bounded by $y=x^2$ and $x=y^2$. [5]

7. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ where $\vec{F}=6z\vec{i}-4\vec{j}+y\vec{k}$ and S is the region of the plane $2x+3y+6z=12$ bounded in the first octant. [5]

8. Evaluate using Gauss divergence theorem, $\iint_S \vec{F}.\vec{n}\,ds$ where $\vec{F}=x^2y\,\vec{i}+xy^2\,\vec{j}+2xyz\vec{k}$ and S is the surface bounded by the planes $x=0$, $y=0$, $z=0$, $x+2y+z=2$. [5]

9. Obtain the Fourier Series to represent $f(x)=x-x^2$ from $x=-\pi$ to $x=\pi$ and deduce that

$$\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\text{.........}$$ [5]

10. Obtain the half range Fourier Sine Series for $f(x)=\pi - x$ in the range $0<x<\pi$. [5]

11. State the conditions for existence of Laplace transform. Obtain the Laplace transform of:

(i) $e^{2t}\cos^3 2t$ (ii) $\frac{\cos 2t-\cos 3t}{t}$ [1+2+2]

12. Find the inverse Laplace transform of:

(i) $\frac{1}{(S-2)(S^2+1)}$ (ii) $\cot^{-1}(S+1)$ [2.5+2.5]

13. Solve the following intial value problem by using Laplace transform:

$y'' + 4y' + 3y = e^t$, $y(0) = 0; y'(0) = 2$ [5]

14. Graphically maximize $Z = 7x_1 + 10x_2$

Subject to constraints:

$$3x_1 + x_2 \le 9$$
$$x_1 + 2x_2 \le 8$$
$$x_1, x_2 \ge 0.$$

[5]

15. Solve the following linear Programming Problem by simple method:

Maximize: $Z = 3x_1 + 5x_2$

Subject to:

$$3x_1 + 2x_2 \le 18$$
$$x_1 \le 4,\ x_2 \le 6$$
$$x_1, x_2 \ge 0.$$

[10]

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01 TRIBHUVAN UNIVERSITY
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Examination Control Division
2073 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Distinguish a matrix and a determinant. Use property of determinant to prove: [5]

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

2. Prove that the necessary and sufficient condition for a square matrix to posses an inverse is that it is non singular. [5]

3. Find the rank of the matrix: [5]

$$\begin{pmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 3 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{pmatrix}$$ by reducing it to normal form.

4. State Cayley-Hamilton theorem and use it to find inverse of the matrix $\begin{pmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{pmatrix}$ [5]

5. Find the work done by the force $\vec{F} = yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k}$ in displacement of a particle along the straight segment C from point (1,1,1) to the point (3,3,2). [5]

6. State Gauss divergence theorem and apply it to evaluate $\iint_S \vec{F}.\vec{n}\,ds$, where $\vec{F} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ and S is the surface of the cube bounded by the planes x = 0, x = a, y = 0, y = a, z = 0, z = a. [5]

7. State and prove Green's theorem in plane. [5]

8. Verify stokes theorem for the vector field $\vec{F} = (2x-y)\,\vec{i} - yz^2\,\vec{j} - y^2z\,\vec{k}$ over the upper half of the surface of $x^2+y^2+z^2=1$ bounded by its projection the xy-plane. [5]

9. Find the Fourier series to represent $f(x) = x - x^2$ from $-\pi$ to π. [5]

10. Find the half range Fourier sine series for $f(x) = e^{2x}$ in $0 < x < \pi$. [5]

11. Define Laplace transform of a function and state criteria of existence of a Laplace transform of a function. Find the Laplace transform of $f(t) = \dfrac{1-\cos 2t}{t}$ [1+1+3]

12. Find inverse Laplace transform of [2+3]

(i) $\frac{1}{s(s+2)}$ (ii) $\tan^{-1}\left(\frac{1}{s}\right)$

13. Solve the following initial value problem using Laplace transform: [5]

$y''+4y'+3y=0,\ \ y(0)=3,\ \ y'(0)=1$

14. Use simplex method to solve the following LPP: [10]

Maximum $z=50x_1+80x_2$

Subject to,

$$x_1+2x_2\le 32$$
$$3x_1+4x_2\le 84$$

15. Graphically maximize [5]

$z=7x_1+10x_2$

Subject to,

$$3x_1+x_2\le 9$$
$$x_1+2x_2\le 8$$
$$x_1,x_2\ge 0$$

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01 TRIBHUVAN UNIVERSITY
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Examination Control Division
2074 Ashwin

Exam.	Back		
Level	BE	Full Marks	80
Programme	ALL (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Use properties of determinant to show [5]

$$\begin{vmatrix} x^2 & x^2-(y-z)^2 & yz \\ y^2 & y^2-(z-x)^2 & zx \\ z^2 & z^2-(x-y)^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)(x^2+y^2+z^2)$$

2. Prove that every square matrix can be uniquely expressed as the sum of symmetric and a skew symmetric matrix. [5]

3. Define eigen values and eigen vectors in terms of linear transformation with matrices as operator. Find eigen values of the matrix. [2+3]

$$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

4. Test the consistency of the system $x+y+z=3$, $x+2y+3z=4$, $2x+3y+4z=7$ by using rank of matrix method and solve if consistent. [5]

5. If $\vec{F}$ is the gradient of some scalar point functions ϕ i.e $\vec{F}=\nabla\phi$, prove that the line integral is independent of the path joining any two points in the region and conversely. [5]

6. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$, where $\vec{F}=xy\,\vec{i}-x^2\,\vec{j}+(x+z)\vec{K}$ and S is the region of the plane $2x+2y+z=6$ bounded in the first quadrant. [5]

7. State and prove Green's theorem in plane. [5]

8. Apply Gauss' divergence theorem to evaluate $\iint_S \left[(x^3-yz)\vec{i}-2x^2y\,\vec{j}+2\vec{K}\right].\vec{n}\,ds$, where S is the surface of the cube bounded by the planes x = 0, x = a, y = 0, y = a, z = 0, z = a. [5]

9. Expand f(x) = x sinx as a Fourier series in $-\pi \le x \le \pi$. [5]

10. Obtain half range cosine series for f(x) = x in the interval $0 \le x \le \pi$. [5]

11. Find the Laplace transform of: [3+2]

   i) $t^2 \cos at$

   ii) $\frac{\sin t}{t}$

12. State convolution theorem for inverse Laplace transform and use it to find the inverse Laplace transform of $\frac{S}{(S^2+4)(S^2+9)}$ [1+4]

13. Solve the following initial value problem by using Laplace transform: [5]

$$y''+2y'-3y=\sin t,\ y(0)=y'(0)=0$$

14. Graphically maximize [5]

$$Z=7x_1+10x_2$$

Subject to constraints,

$$3x_1+x_2\le 9$$
$$x_1+2x_2\le 8$$
$$x_1,x_2\ge 0$$

15. Solve the following LPP by simplex method using duality of: [10]

Minimize $Z=20x+50y$

Subject to:

$$2x+5y\ge 12$$
$$3x+7y\ge 17$$
$$x,y\ge 0$$

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01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2072 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Use properties of determinants to prove: [5]

$$\begin{vmatrix} a^2+1 & ba & ca & da \\ ab & b^2+1 & cb & db \\ ac & bc & c^2+1 & dc \\ ad & bd & cd & d^2+1 \end{vmatrix} = 1+a^2+b^2+c^2+d^2$$

2. Show that every square matrix can be uniquely expressed as the sum of symmetric and Skew-Symmetric matrices. [5]

3. Test the consistency of the system $x+y+z=3, x+2y+3z=4$ and $2x+3y+4z=7$ and solve completely if found consistent. [5]

4. State Cayley-Hamilton theorem and verify it for the matrix; $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$ [1+4]

5. Prove that " The line integral $\int_c \vec{F}.d\vec{r}$ of a continuous function $\vec{F}$ defined in a region R is independent of path C joining any two points in R if and only if there exists a single valued scalar function ϕ having first order partial derivatives such that $\vec{F} = \nabla\phi$". [5]

6. State Green's theorem and use it to find the area of astroid $x^{2/3} + y^{2/3} = a^{2/3}$ [5]

7. Evaluate $\iint_s \vec{F}.\vec{n}\,ds$, where $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and 's' is the surface of the plane $x + y + z = 1$ between the co-ordinate planes. [5]

8. Apply Gauss' divergence theorem to evaluate $\iint_s \vec{F}.\vec{n}\,ds$ where

   $\vec{F} = (x^3 - yz)\vec{i} - 2x^2y\,\vec{j} + 2\vec{k}$ and 's' is the surface the cube bounded by the planes $x = 0, x = a, y = 0, y = a, z = 0, z = a$. [5]

9. Find the Laplace transform of: [2+3]

i) $t\text{Sin}^2 3t$

ii) $\frac{\text{Sin}2t}{t}$

10. Find the inverse Laplace transform of: [2+3]

i) $\frac{1}{s^2 - 3s + 2}$

ii) $\frac{1}{s(s+1)^3}$

11. Apply Laplace transform to solve the differential equation: [5]

$y'' + 2y' + 5y = e^{-t}\sin t, \quad x(0) = 0, x'(0) = 1$

12. Find a Fourier series to represent $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that

$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}$ [5]

13. Develop $f(x) = \sin\left(\frac{\pi x}{l}\right)$ in half range Cosine Series in the range $0 < x < l$. [5]

14. Graphically maximize, [5]

$Z = 7x_1 + 10x_2$

Subject to constraints,

$3x_1 + x_2 \leq 9$

$x_1 + 2x_2 \leq 8$

$x_1 \geq 0, x_2 \geq 0$

15. Solve the following LPP using simplex method. [10]

Maximize: $P = 50x_1 + 80x_2$

Subject to: $x_1 + 2x_2 \leq 32$

$3x_1 + 4x_2 \leq 84$

$x_1 \geq 0, x_2 \geq 0$

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01 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2073 Shrawan

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	ALL (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt* ***All*** *questions.*
- ✓ *The figures in the margin indicate* ***Full Marks***.
- ✓ *Assume suitable data if necessary.*

1. Use properties of determinants to prove: $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$ [5]

2. Prove that the necessary and sufficient condition for a square matrix A to posses an inverse is that the matrix A should be non singular. [5]

3. Find the rank of the matrix $\begin{pmatrix} 1 & 3 & -2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{pmatrix}$ [5]

   by reducing it into normal form.

4. Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ [4+1]

   Give an example showing importance of eigenvectors.

5. Show that $\vec{F} = (2x+z^2)\vec{i} + Z\vec{j} + (y+2xz)\vec{K}$ is irrotational and find its scalar potential. [5]

6. State and prove Green's Theorem in plane. [5]

7. Evaluate $\iint_S \vec{F}.\vec{n}\, ds$, where $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and S is the surface of the sphere $x^2+y^2+z^2=1$ in the first octant. [5]

8. Evaluate $\int_c xydx + xy^2dy$ by applying stokes theorem where C is the square in xy-plane with vertices (1,0), (-1,0), (0,1), (0,-1) [5]

9. Find the Laplace transform of : [2+3]

   i) $te^{2t}\sin 3t$

   ii) $\frac{e^{-t}\sin t}{t}$

10. Find the inverse Laplace transform of : [2+3]

i) $\dfrac{s+2}{s^2-4s+13}$

ii) $\log\left(\dfrac{s+a}{s-a}\right)$

11. Solve the following initial value problem using Laplace transform: [5]

$x''+4x'+4x=6e^{-t}, \quad x(0)=-2, \quad x'(0)=-8$

12. Find the Fourier series representation of $f(x)=|x|$ in $[-\pi, \pi]$ [5]

13. Obtain the half range Fourier Sine Series for the function $f(x) = x^2$ in the interval (0, 3). [5]

14. Apply Graphical method to maximize, [5]

$Z=5x_1+3x_2$

Subject to the constraints:

$x_1+2x_2 \le 50$

$2x_1+x_2 \le 40$

$x_1 \ge 0, \ x_2 \ge 0$

15. Solve the following Linear Programming Problem by Simplex method: [10]

Maximize: $Z=15x_1+10x_2$

Subject to: $x_1+3x_2 \le 10$

$2x_1+x_2 \le 10$

$x_1 \ge 0, \ x_2 \ge 0$

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01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2071 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt __All__ questions.*
- ✓ *The figures in the margin indicate __Full Marks__.*
- ✓ *Assume suitable data if necessary.*

1. Using the properties, evaluate the determinant: [5]

$$\begin{vmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b^2 & b^3+cda \\ 1 & c & c^2 & c^3+abd \\ 1 & d & d^2 & d^3+abc \end{vmatrix}$$

2. Prove that every square matrix can uniquely be expressed as the sum of a symmetric and a skew symmetric matrix. [5]

3. Test the consistency of the system: [5]

$$x-6y-z=10,\ 2x-2y+3z=10,\ 3x-8y+2z=20$$

And solve completely, if found consistent.

4. Find the eigen values and eigenvecters of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$. [5]

5. Using the line integral, compute the workdone by the force [5]

$$\vec{F}=(2x-y+2z)\vec{i}+(x+y-z)\vec{j}+(3x-2y-5z)\vec{k}$$

when it moves once around a circle $x^2+y^2=4; z=0$

6. State and prove Green's Theorem in plane. [5]

7. Verify Stoke's theorem for $\vec{F}=(x^2+y^2)\vec{i}-2xy\vec{j}$ taken around the rectangle bounded by the lines $x=\pm a$, $y=0$, $y=b$. [5]

8. Evaluate $\iint_S \vec{F}.\vec{n}\, ds$ where $\vec{F}=(2xy+z)\vec{i}+y^2\vec{j}-(x+3y)\vec{K}$ by Gauss divergence theorem; where S is surface of the plane $2x+2y+z=6$ in the first octant bounding the volume V. [5]

9. Find the Laplace transform of the following: [2.5×2]

   a) $te^{-2t}\cos t$

   b) $\text{Sinhat}.\cos t$

10. Find the inverse Laplace transform of : [2.5×2]

a) $\dfrac{1}{S(S+1)}$

b) $\dfrac{S^2}{(S^2+b^2)^2}$

11. Solve the differential equation $y''+2y'+5y = e^{-t}\sin t, y(0)=0, y'(0)=1$, by using Laplace transform. [5]

12. Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \le x \le \pi$. [5]

13. Obtain half range sine series for the function $f(x) = x - x^2$ for $0 < x < 1$. [5]

14. Graphically maximize and minimize [5]

$z = 9x + 40y$ subjected to the constraints

$y - x \ge 1, y - x \le 3, 2 \le x \le 5$

15. Solve the following Linear Programming Problem by Simplex method: [10]

Maximize, $P = 20x_2 - 5x_1$

Subjected to, $10x_2 - 2x_1 \le 5$

$2x_1 + 5x_2 \le 10$ and $x_1, x_2 \ge 0$

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01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2072 Kartik

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Prove that $\begin{vmatrix} (a+b)^2 & ca & bc \\ ca & (b+c)^2 & ab \\ bc & ab & (c+a)^2 \end{vmatrix} = 2abc(a+b+c)^3$ [5]

2. If A and B are two non singular matrices, then prove that $(AB)^{-1} = B^{-1}A^{-1}$ [5]

3. Find the rank of the matrix: [5]

$$\begin{pmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

4. Find the eigen values and eigen vectors of the matrix. [5]

$$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

5. Prove that the line integral $\int_A^B \vec{F}.d\vec{r}$ is independent of path joining any two points A and B in the region R, if and only if, $\int_C \vec{F}.d\vec{r} = 0$ for any simple closed path C in R. [5]

6. Evaluate $\iint_S \vec{F}.\vec{n}\,ds$ where $\vec{F} = yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. [5]

OR

Apply Stoke's theorem to evaluate $\int_C (x+y)dx + (2x-z)dy + (y+z)\,dz$ where C is the boundary of the triangle with vertices (2,0,0), (0,3,0) and (0,0,6). [5]

7. State Green's theorem in plane and hence apply it to compute the area of the curve $x^{2/3} + y^{2/3} = a^{2/3}$. [5]

8. Apply Gauss divergence theorem to evaluate $\iint_S \vec{F}.\vec{n}\, ds$ where $\vec{F} = x^2\vec{i} + z\vec{j} + yz\vec{k}$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. [5]

9. Find the Laplace transform of the following: [2.5×2]

   a) $\dfrac{\cos 2t - \cos 3t}{t}$

   b) $\sin^3 2t$

10. Find the inverse Laplace transform of the following: [2+3]

   a) $\dfrac{1}{s^2 - 5s + 6}$

   b) $\dfrac{s+2}{(s^2 + 4s + 5)^2}$

11. Solve the initial value problem by using Laplace transform: [5]

   $x'' + 2x' + 5x = e^{-t}\sin t;\ x(0) = 0,\ x'(0) = 1$

12. Obtain Fourier Series for the function $f(x) = x - x^2$ from $-\pi$ to π and hence show that: [5]

   $$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\dots\dots$$

13. Obtain the half range sine series for the function $f(x) = x^2$ in the interval (0,3). [5]

14. Graphically maximize and minimize [5]

   $Z = 5x_1 + 3x_2$ Subjected to constraints

   $3x_1 + 5x_2 \le 15$

   $5x_1 + 2x_2 \le 10, x_1, x_2 \ge 0$

15. Use simplex method to solve the Linear Programming problem: [10]

   Maximize $Z = 15x_1 + 10x_2$

   Subject to $2x_1 + 2x_2 \le 10$

   $x_1 + 3x_2 \le 10$

   and $x_1, x_2 \ge 0$

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01 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2070 Chaitra

Exam.	Regular		
Level	BE	**Full Marks**	80
Programme	All (Except B.Arch)	**Pass Marks**	32
Year / Part	II / I	**Time**	3 hrs.

Subject: **- Mathematics III *(SH501)***

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Using the properties of determinant prove [5]

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

2. Prove that $(AB)^T = B^T A^T$ where A is the matrix of size m×p and B is the matrix of size p×n [5]

3. Find the rank of the following matrix by reducing normal form. $\begin{bmatrix} 1 & 3 & -2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{bmatrix}$ [5]

4. Find the eigen values and eigen vectors of the following matrix. $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$ [5]

5. Prove that the line intergral $\int_A^B \vec{F}.d\vec{r}$ is independent of the path joining any two points A and B in a region if $\int_c \vec{F}.d\vec{r} = 0$ for any simple closed curve C in the region. [5]

6. Evaluate $\iint_s \vec{F}.\hat{n}\ ds$ where $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and S is the finite plane x + y + z = 1 between the coordinate planes. [5]

OR

Evaluate $\iint_s \vec{F}.\hat{n}\,ds$ for $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ where S is the surface of sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

7. Evaluate, $\iint_s \vec{F}.\hat{n}\,ds$ for $\vec{F} = x\vec{i} - y\vec{j} + (z^2 - 1)\vec{k}$ where S is the surface bounded by the cylinder $x^2 + y^2 = 4$ and the planes z = 0 and z = 1 [5]

8. Verify the stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the upper part of the sphere $x^2 + y^2 + z^2 = a^2$ C is its boundary. [5]

9. Find the Laplace transform of (a) $t^2 \sin zt$ and (b) $\frac{1-e^t}{t}$ [2.5×2]

10. Find the inverse Laplace transform of (a) $\frac{2s+3}{s^2+5s-6}$ (b) $\frac{s^3}{s^4-a^4}$ [2.5×2]

11. Solve the following differential equation by using Laplace transform [5]

$y'' + y' - 2y = x$, $y(0) = 1$, $y'(0) = 0$

12. Obtain the Fourior series for $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence prove that

$$\sum \frac{1}{x^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \ldots\ldots\ldots = \frac{\pi^2}{6}$$ [5]

13. Obtain half range sine series for $f(x) = \pi x - x^2$ in $(0, \pi)$ [5]

14. Graphically minimize $z = 4x_1 + 3x_2 + x_3$ [5]

Subject to $x_1 + 2x_2 + 4x_3 \geq 12$

$3x_1 + 2x_2 + x_3 \geq 8$ and $x_1, x_2, x_3 \geq 0$

15. Minimize $z = 8x_1 + 9x_2$ [10]

Subject to $x_1 + 3x_2 \geq 4$

$2x_1 + x_2 \geq 5$ with $x_1, x_2 \geq 0$

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01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2071 Shawan

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Show that: $\begin{vmatrix} (b+c)^2 & b^2 & c^2 \\ a^2 & (c+a)^2 & c^2 \\ a^2 & b^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$ [5]

2. Prove that every square matrix can be uniquely written as a sum of Hermitian and Skew-Hermitian matrices. [5]

3. Find the rank of the matrix by changing it into normal form: $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \\ 1 & 2 & 4 \end{pmatrix}$ [5]

4. Find the eigen value and eigen vector of the matrix: $\begin{pmatrix} 2 & 1 & 1 \\ -2 & 1 & 3 \\ 2 & 1 & -1 \end{pmatrix}$ [5]

5. Using Green's theorem, evaluate $\int_C (y^3dx - x^3dy)$ where C is the boundary of the circle $x^2 + y^2 = 4$. [5]

6. Show that $\vec{F}(x,y,z) = y^3\,\vec{i} + (3xy^2 + e^{2z})\,\vec{j} + 2ye^{2z}\,\vec{k}$ is conservative vector field and find its scalar potential function. [5]

7. Find the surface integral $\iint_S \vec{F}.\hat{n}\,ds$ where $\vec{F} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$. [5]

8. Verify Stoke's theorem for $\vec{F}(x,y,z) = (2x - y)\,\vec{i} - yz^2\,\vec{j} - y^2z\,\vec{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 4$ and C is its boundary. [5]

OR

Evaluate using Gauss divergence theorem,
$\iint_S \vec{F}.\hat{n}\,ds$ where $\vec{F}(x,y,z) = x^2y\,\vec{i} + xy^2\,\vec{j} + 2xyz\,\vec{k}$ and S is the surface bounded by the planes x = 0, y = 0, z = 0 and x+2y+z = 2

9. Find the Laplace transform of (i) sin 2t cosh 4t (ii) $te^{2t}\sin 4t$. [5]

10. Using the Convolution theorem, find the inverse Laplace transform of $\dfrac{3s}{(s^2+4)(s^2+1)}$ [5]

11. Solve the following initial value problem using Laplace transform: [5]

$y''+4y'+3y = e^t, y(0) = 00, y'(0) = 2$

12. Obtain the half range Fourier sine series of $f(x) = \pi - x$ in the range $0 < x < \pi$. [5]

13. Obtain the Fourier series of $f(x) = e^{3x}$ in $0 < x < 2\pi$. [5]

14. Graphically maximum $Z = 5x_1 + 3x_2$ subject to constraints [5]

$x_1 + 2x_2 \leq 50, 2x_1 + x_2 \leq 40$ and $x_1 \geq 0, x_2 \geq 0$

15. Solve the following linear programming problem by simplex method constructing the duality: [10]

Minimize: $P = 21x_1+50x_2$

Subject to $3x_1+7x_2 \geq 17$

$2x_1+5x_2 \geq 12$

$x_1, x_2 \geq 0$

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02 TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2070 Ashad

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B. Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Prove that: $\begin{vmatrix} a & b & b & b \\ a & b & a & a \\ a & a & b & a \\ b & b & b & a \end{vmatrix} = -(b-a)^4$ [5]

2. Prove that every matrix A can uniquely be expressed as a sum of a symmetric and a skew symmetric matrix. [5]

3. Test the consistency of the system x+y+z = 3, x+2y+3z = 4 and 2x+3y+4z = 7 and solve if consistent. [5]

4. Verify Cayley-Hamilton theorem for matrix A and find the inverse of $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ [5]

5. State and prove Green's theorem in the plane. [5]

OR

Verify Stroke's theorem for $\vec{F} = (x^2+y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle in the xy-plane bounded by x = 0, x = a, y =0, y = b

6. Find the work done in moving particle once round the circle $x^2+y^2=9,\ z=0$ under the force field $\vec{F}$ given by $\vec{F} = (2x-y+z)\vec{i} + (x+y-z^2)\vec{j} + (3x-2y+4z)\vec{k}$ [5]

7. Evaluate $\iint_s \vec{F}.\vec{n}\ ds$ where $\vec{F} = xy\vec{i} - x^2\vec{j} + (x+z)\vec{k}$, s is the portion of the plane 2x+2y+z = 6 included in the first octant. [5]

8. Show that $\iint_s \left[(x^3-yz)\vec{i} - 2x^2y\vec{j} + 2\vec{k}\right].\vec{n}\ ds = \frac{a^5}{3}$ where s is the surface of the cube bounded by the planes x = 0, x = a, y = 0, y = a, z = 0, z = a [5]

9. Find the Laplace transform of (i) $f(t) = \frac{1-\cos t}{t}$ (ii) $f(t) = te^{-t}\sin t$ [5]

10. Find the inverse Laplace transform of (i) $\frac{(s+2)^3}{s^4}$ (ii) $\frac{1}{s^2(s^2+a^2)}$ [5]

11. Using Laplace Transform to solve: $y''+4y = \sin t$; $y(0) = 0 = y'(0)$ [5]

12. Find a fourier series to represent $f(x) = x - x^2$ from $x = -\Pi$ to $x = \Pi$ [5]

13. Find a fourier series to represent $f(x) = 2x - x^2$ in the range (0,3) [5]

OR

Express $f(x) = x$ as a half range sine series in $0 < x < \Pi$

14. Use simplex method to, Maximize $p = 15x_1+10x_2$ [7]

Subject to $2x_1 + x_2 \le 10$

$x_1 + 3x_2 \le 10$, $x_1, x_2 \ge 0$

15. Find the dual of following Linear programming problem and solve by simplex method [8]

Minimize $C = 16x_1 + 45x_2$

Subject to $2x_1 + 5x_2 \ge 50$

$x_1 + 3x_2 \ge 27$, $x_1,x_2 \ge 0$

OR

Use Big M-method to solve the following linear programming problem.

Maximize $p = 2x_1 + x_2$

Subject to $x_1 + x_2 \le 10$

$-x_1 + x_2 \ge 2$, $x_1, x_2 \ge 0$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2069 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *The figures in the margin indicate Full Marks.*
- ✓ *Assume suitable data if necessary.*

1. Find the value of the determinant $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$ [5]

2. Show that the matrix $B^\theta AB$ is Hermitian or skew-Hermittian according as A is Hermitian and skew- Hermitian. [5]

3. Find the rank of the matrix $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ reducing this into the triangular form. [5]

4. Obtain the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and verify that it is satisfied by A. [5]

5. Evaluate $\int_c \vec{F}.\vec{dr}$, where $\vec{F} = (x-y)\vec{i} + (x+y)\vec{j}$ along the closed curve C bounded by $y^2 = x$ and $x^2 = y$ [5]

6. Find the value of the normal surface integral $\iint_S \vec{F}.\vec{n}\,ds$ for $\vec{F} = x\vec{i} - y\vec{j} + (z^2-1)\vec{k}$, where S is the surface bounded by the cylinder $x^2 + y^2 = 4$ between the planes $Z = 0$ and $Z = 1$. [5]

7. Using Green's theorem, find the area of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ [5]

8. Verify stoke's theorem for $\vec{F} = 2y\vec{i} + 3x\vec{j} - z^2\vec{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 9$ and C is its boundary. [5]

OR

Evaluate the volume intergral $\iiint_V \vec{F}\,dv$, where V is the region bounded by the surface $x = 0, y = 0, y = 6, z = x^2, z = 4$ and $\vec{F} = 2xz\,\vec{i} - x\,\vec{j} + y^2\,\vec{k}$

9. Find the Laplace transforms of the following functions [2.5×2]
   a) $t\,e^{-4t}\sin 3t$
   b) $\dfrac{\cos at - \cos bt}{t}$

10. State and prove the second shifting theorem of the Laplace transform. [5]

11. Solve the following differential equation using Laplace transform. [5]

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=x \text{ given } y(0)=1, y'(0)=0$$

12. Obtain the Fourier series for $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence show that

$$\sum\frac{1}{n^2}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots=\frac{\pi^2}{6}$$ [5]

13. Express $f(x) = x$ as a half-range sine series in $0 < x < 2$ [5]

14. Maximize $Z = 4x_1+5x_2$ subject to constraints [5]

$2x_1 + 5x_2 \leq 25$
$6x_1 + 5x_2 \leq 45$
$x_1 \geq 0$ and $x_2 \geq 0$

graphically

15. Solve the following linear programming problem using the simplex method. [10]

Maximize $P = 50x_1+80x_2$
Subject to $x_1+ 2x_2 \leq 32$
$3x_1 + 4x_2 \leq 84$
$x_1, x_2 \geq 0$

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02 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division

2068 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BCE, BEL, BEX, BCT, BME, BIE, B. AGRI.	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Engineering Mathematics III *(SH 501)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. Prove that: $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$. [5]

2. Define Hermition and Skew Hermition matrix. Show that every square matrix can be uniquely expressed as the sum of a Hermition and a skew Hermition. [5]

3. For what value of λ the equation $x + y + z = 1$, $x + 4y + 10z = \lambda^2$ and $x + 2y + 4z = \lambda$ have a solution? Solve them completely in each case. [5]

4. Find the eigen values and eigen vectors of $A = \begin{vmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{vmatrix}$. [5]

5. Evaluate $\int_C \vec{F}.\vec{dr}$, Where C: $x^2 = y$ and $y^2 = x$ and $\vec{F} = (x-y)\vec{i} + (x+y)\vec{j}$. [5]

6. State and prove Green theorem in a plane. [5]

7. Verify Guess divergence theorem for $\vec{F} = x^2\vec{i} + 3\vec{j} + yz\vec{k}$. Taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z= 0, z = 1$. [5]

8. Find the Laplace transform of the given function (i) $t^2\sin t$ (ii) $\cos at \sinh at$. [5]

9. Evaluate $\iint_s \vec{F}.\hat{n}ds$ where $\vec{F} = 3\vec{i} + x\vec{j} - yz\vec{k}$ and s is the surface of the cylinder $x^2 + y^2 = 9$ included in the first octant between the plane $z = 0, z = 4$. [5]

10. Find the inverse Laplace transform: (a) $\frac{1}{(S-2)(S+4)}$ (b) $\log\left(\frac{s^2+a^2}{s^2}\right)$. [5]

11. Solve the equation using Laplace transform $y'' + 4y' + 3y = t$, $t>0$ $y(0) = 0$, $y'(0) = 1$. [5]

12. Obtain a Fourier series to represent the function $f(x) = |x|$ for $-\pi \le x \le \pi$ and hence deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots$ [5]

13. Obtain the half Range Sine Series $f(x) = e^{x}$ in $0<x<1$. [5]

OR

Obtain the Fourier series for $f(x) = x - x^2$ where $-1 < x < 1$ as a Fourier series of period 2.

14. Solve the following by using the simplex method: [7.5]

Maximize $P = 15x_1 + 10x_2$,
Subject to
$2x_1 + x_2 \le 10$,
$x_1 + 3x_2 \le 10$,
$x_1, x_2 \ge 0$.

15. Solve by using the dual method: [7.5]

Minimize $C = 21x_1 + 50x_2$,
Subject to $2x_1 + 5x_2 \ge 12$,
$3x_1 + 7x_2 \ge 17$,
$x_1, x_2 \ge 0$.

OR

Solve the following LPP by using the big M-method:

Maximize $P = 2x_1 + x_2$,
Subject to
$x_1 + x_2 \le 10$,
$-x_1 + x_2 \ge 2$,
$x_1, x_2 \ge 0$.

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COMPUTER II / I

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2067 Ashadh

Exam.	Regular/Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Mathematics III

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Using the properties of determinant prove:

$$\begin{vmatrix} a^2+1 & ba & ca & da \\ ab & b^2+1 & cb & db \\ ac & bc & c^2+1 & dc \\ ad & bd & cd & d^2+1 \end{vmatrix} = a^2+b^2+c^2+d^2+1$$

2. Show that every square matrix can be uniquely expressed as the sum of hermitian and a skew-hermitian matrix.

3. Reduce to normal form and find the rank of the matrix:

$$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

4. Find the eigen values and eigne vectors of the matrix

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

5. Find the Laplace transform of:

   a) coshat sin at b) $\dfrac{\cos 2t - \cos 3t}{t}$

6. Find the inverse Laplace transform of:

   a) $\dfrac{1}{s^2(s^2+a^2)}$ b) $\log \dfrac{s+1}{s-1}$

7. State and prove the integral theorem of the Laplace transform.

8. Solve the following differential equation using the Laplace transform.

   $y''' + 2y'' - y' - 2y = 0$ where $y(0) = y'(0) = 0$ and $y''(0) = 6$

9. Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to π. Hence show that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

10. Express $f(x) = x$ as a cosine half range series in $0 < x < 2$.

11. The acceleration of a moving particle at any time t is given by $\frac{d^2 \vec{r}}{dt^2} = 12\cos 2t\hat{i} - 8\sin 2t\hat{j} + 16t\hat{k}$. Find the velocity $\vec{v}$ and displacement $\vec{r}$ at anytime t if $t = 0$, $\vec{v} = 0$ and $\vec{r} = 0$.

12. Find the angle between the normals to the surface $xy = z^2$ at the points $(1,4,2)$ and $(-3,-3,3)$.

13. Find the work done in moving a particle once round the circle $x^2 + y^2 = 9, z = 0$ under the force field $\vec{F}$ given by $\vec{F} = (2x - y + z)\vec{i} + (x + y - z^2)\vec{j} + (3x - 2y + 4z)\vec{k}$.

14. Evaluate $\iint_s \vec{F}\cdot\vec{n}\, ds$ where s is the upper side of triangle with vertices $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ where $\vec{F} = (x - 2z)\vec{i} + (x + 3y + z)\vec{j} + (5x + y)\vec{k}$.

15. State Green's theorem in a plane. Using Green's theorem find the area of $x^{2/3} + y^{2/3} = a^{2/3}$.

16. Verify Stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where s is the upper part of the sphere $x^2 + y^2 + z^2 = a^2$ and c is its boundary.

OR

Verify Gauss theorem for $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ over the region bounded by $x^2 + y^2 = a^2$, $z = 0$ and $z = h$.

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05 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2066 Bhadra

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	II / I	Time	3 hrs.

Subject: - Mathematics III

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *All questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. Find the value of the determinant without expansion.

$$\begin{vmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix}$$

2. Show that the necessary and sufficient condition for a square matrix A to posses inverse is that A is non-singular.

3. Reduce to normal form and hence find the rank of the matrix.

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 4 & 2 & -8 \end{bmatrix}$$

4. Verify Cayley-Hamilton theorem for a matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

5. Find the Laplace transforms of (i) $t\, e^{-4t} \sin 3t$ (ii) coshat sinat

6. Find the inverse Laplace transform of (i) $\log \frac{s+1}{s-1}$ (ii) $\frac{s^2}{(s-3)(s^2+4)}$

7. If $\mathcal{L}\{f(t)\} = F(s)$, the prove that $\mathcal{L}\left\{\int_0^t f(u)du\right\} = \frac{1}{s}F(s),\ s > 0.$

8. Using Laplace transform solve the equation
$(D^2 + 5D + 6)x = 5e^t$
given $x(0) = 2,\ x'(0) = 1$

9. If $f(x) = x^2$ and $f(x + 2\pi) = f(x)$ in $[-\pi, \pi]$, find the Fourier series expansion of x^2 in the interval $-\pi \le x \le \pi$. Also deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots\dots = \frac{\pi^2}{6}$.

10. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, −1, 2).

11. Find the work done in moving a particle once round the circle $x^2 + y^2 = 9$, $z = 0$ under the force field $\vec{F}$ given by $\vec{F} = (2x - y + z)\,\vec{i} + (x + y - z^2)\,\vec{j} + (3x - 2y + 4z)\,\vec{k}$.

12. Prove that the line integral $\int_A^B \vec{F}.d\vec{r}$ is independent of the path joining any two points A and B in a region R if and only if $\int_C \vec{F}.d\vec{r} = 0$ for any simple closed curve C in R.

13. Evaluate $\iint_S \vec{F}.\vec{n}\, ds$ where $\vec{F} = xy\,\vec{i} - x^2.\vec{j} + (x + z)\,\vec{k}$ and s in the portion of the plane $2x + 2y + z = 6$ included in the first octant.

OR

State and prove Green's theorem in the plane.

14. Verify Stoke's theorem for $\vec{F} = 2y\,\vec{i} + 3x\,\vec{j} - z^2\,\vec{k}$ where s is the upper half of the sphere $x^2 + y^2 + z^2 = 9$ and C its boundary.

OR

Verify gauss theorem for $\vec{F} = y\,\vec{i} + x\,\vec{j} + z^2\,\vec{k}$ over the region bounded by $x^2 + y^2 = a^2$, $z = 0$ and $z = h$.

15. Obtain the half-range sine series for e^x in $0 < x < 1$.

16. Find the Fourier series of the function: $f(x) = x^3$ $(-\pi < x < \pi)$.

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