

Exam.	Regular		
Level	BE	Full Marks	80
Programme	ALL (Except B. Arch)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I (SH401)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. State Leibnitz theorem. If $\log y = \tan^{-1} x$, then show that

$$(1+x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + (n^2 + n)y_n = 0 \quad [1+4]$$

2. State Rolle's theorem. Is the theorem true when the function is not continuous at the end points? Justify your answer. Verify Rolle's theorem for $f(x) = x^2 5x + 6$ on $[2,3]$. [1+2+2]

3. State L-Hospital's rule. Evaluate $\lim_{x \rightarrow 1} (2-x)^{\tan\left(\frac{\pi x}{2}\right)}$ [1+4]

4. Find the asymptotes of the curve $(x+y)^2(x+2y+2) = x+9y-2$ [5]

5. Find the pedal equation of the ellipse $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$. [5]

6. Evaluate the integral $\int_{-1}^1 \frac{1}{x^2} dx$ [5]

7. Apply the rule of differentiation under integral sign to evaluate $\int_0^{\infty} \frac{e^{-ax} \sin x}{x} dx$ and hence

deduce that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$ [5]

8. Define Beta function. Apply Beta and Gamma function to evaluate $\int_0^{2a} x^5 \sqrt{2ax - x^2} dx$ [5]

9. Find the area common to the circle $r = a$ and the cardioid $r = a(1+\cos\theta)$ [5]

10. Through what angle should the axes be rotated to reduce the equation

$$3x^2 + 2xy + 3y^2 - \sqrt{2}x = 0$$

into one with the xy term missing? Also obtain the transformed equation. [2+3]

11. Derive the equation of an ellipse in standard form. [5]

12. Find the product of semi-axis of the conic $x^2 - 4xy + 5y^2 = 2$ [5]

OR

Describe and sketch the graph of conic $r = \frac{12}{3+2\cos\theta}$

13. Solve the differential equation of $(x^2 - y^2)dx + 2xydy = 0$ [5]

14. Solve: $y = yp^2 + 2px$ where $p = \frac{dy}{dx}$ [5]

15. Solve $(D^2 - 6D + 9)y = x^2 e^{2x}$ [5]

Exam.	Back		
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- ✓ Attempt All questions.
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1. State Leibnitz's theorem on higher order derivative. If $y = e^{a \tan^{-1} x}$, prove that $(1+x^2)y_{n+2} + (2nx + 2x - a)y_{n+1} + n(n+1)y_n = 0$
2. State difference between Roll's Theorem and Lagrange's Mean value theorem. Verify Lagrange's mean value theorem for $f(x) = x(x-1)(x-2)$ when $x \in \left[0, \frac{1}{2}\right]$.
3. Define indeterminate form of a function. Evaluate

$$\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x^2}$$

4. Define asymptote to a curve. Find the asymptotes of curve $y^3 + 2xy^2 + x^2y - y + 1 = 0$.
5. Find radius of curvature of the curve $x^3 + y^3 = 3axy$ at origin.

OR

Find the pedal equation of the polar curve $r^m = a^m \cos m\theta$.

6. Integrate : $\int_0^{\pi/2} \frac{\cos x \, dx}{(1 + \sin x)(2 + \sin x)}$
7. Apply differentiation under integral sign to evaluate $\int_0^{\infty} \frac{e^{-ax} \sin x}{x} dx$.
8. Define Beta and Gamma function. Use them to evaluate $\int_0^{2a} x^5 \sqrt{2ax - x^2} dx$.
9. Show that the area of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is $\frac{3}{8} \pi a^2$.

OR

Find the volume of the solid formed by the revolution of the cardioid $r = a(1 + \cos\theta)$ about the initial line.

10. Solve: $(1 + y^2) dx = (\tan^{-1} y - x) dy$
11. Solve: $y = px - \sqrt{m^2 + p^2}$ where $p = \frac{dy}{dx}$.

12. Solve: $(D^2 + 2D + 1)y = e^x + x^2$.

13. Solve: $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$.

OR

A resistance of 100 ohms, an inductance of 0.5 Henry are connected in series with a battery of 20 volts. Find the current in the circuit as a function of time.

14. What does the equation of lines $7x^2 + 4xy + 4y^2 = 0$ become when the axes are the bisectors of the angles between them?

15. Derive the equation of hyperbola in standard form.

16. Find the foci and eccentricity of the conic $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$.

OR

Describe and sketch the graph of the conic $r = \frac{12}{6 + 2\sin\theta}$.

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- ✓ Attempt **All** questions.
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1. State Leibnitz's theorem. If $y = (\sin^{-1} x)^2$, show that
 $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$

2. Verify Rolle's Theorem for $f(x) = \log \frac{x^2 + ab}{(a+b)x}$; $x \in [a, b]$. How does Rolle's Theorem differ from Lagrange's mean value theorem.

3. Evaluate $\lim_{x \rightarrow 0^+} \left(\frac{\sin x}{x} \right)^{\frac{1}{x}}$

4. Find the asymptotes to the curve $y^3 + 2xy^2 + x^2y - y + 1 = 0$

5. Find the radius of curvature at origin for the curve $x^3 + y^3 = 3axy$.

6. Show that $\int_0^{\pi} x \log(\sin x) dx = \frac{\pi^2}{2} \log \frac{1}{2}$

7. Apply the rule of differentiation under integral sign to evaluate $\int_0^{\infty} \frac{e^{-ax} \sin x}{x} dx$ and hence

deduce that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$

8. Define Beta function. Apply Beta and Gamma function to evaluate $\int_0^{2a} x^5 \sqrt{2ax - x^2} dx$

9. Find the volume generated by revolution of astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about x-axis.

10. What does the equation $3x^2 + 3y^2 + 2xy = 2$ becomes when the axes are turned through an angle of 45° to the original axes?

11. Find center, length of axes, eccentricity and directrices of the conic

$$3x^2 + 8xy - 3y^2 - 40x - 20y + 50 = 0$$

OR

Describe and sketch the conic $r = \frac{12}{2 - 6\cos\theta}$

12. Deduce standard equation of ellipse.

13. Solve the differential equation: $(1+y^2) + (x - e^{\tan^{-1}y}) \frac{dy}{dx} = 0$

14. Solve: $xp^2 - 2yp + ax = 0$ where $p = \frac{dy}{dx}$

15. Solve: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x} \cdot \sin x$

16. Resistance of 100 ohms, an inductance of 0.5 Henry are connected in series with battery

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1. State Leibnitz's theorem. If $y = (x^2 - 1)^n$, then prove that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n-1)y_n = 0$$

2. Assuming the validity of expansion, expand $\log(1 + \sin x)$ by Maclaurin's theorem.

3. Evaluate $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x}$

4. Find the asymptotes of the curve: $x(x-y)^2 - 3(x^2 - y^2) + 8y = 0$

5. Find the radius of curvature at any point (r, θ) for the curve $a^2 = r^2 \cos 2\theta$

6. Show that: $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx = \frac{\pi^2}{4}$

7. Apply differentiation under integral sign to evaluate $\int_0^{\pi/2} \log \frac{a + b \sin x}{a - b \sin x} dx$

8. Define Gamma function. Apply Beta and Gamma function to evaluate:

$$\int_0^{\pi/6} \cos^2 6\theta \cdot \sin^4 3\theta = \frac{7\pi}{192}$$

9. Find the area enclosed by $y^2(a-x) = x^3$ and its asymptotes.

10. If the axes be turned through an angle of $\tan^{-1} 2$, what does the equation $4xy - 3x^2 - a^2 = 0$ become?

11. Find the center, length of axes, eccentricity and directrices of the conic.

$$2x^2 + 3y^2 - 4x - 12y + 13 = 0$$

OR

Describe and sketch the graph of the conic $r = \frac{10}{3 + 2 \cos \theta}$

12. Deduce standard equation of hyperbola.

13. Solve the differential equation: $x \log x \frac{dy}{dx} + y = 2 \log x$

14. Solve: $(x-a)p^2 + (x-y)p - y = 0$; where $p = \frac{dy}{dx}$

15. Solve: $(D^2 - D - 2)y = e^x + \sin 2x$

16. Find a current $i(t)$ in the RLC circuit assuming zero initial current and charge q , if $R = 80$ ohms, $L = 20$ Henry, $C = 0.01$ Faradays and $E = 100$ volts.

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1. If $y = (\sin^{-1} x)^2$ then show that:

i) $(1-x^2)y_2 - xy_1 - 2 = 0$

ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$

2. State Rolle's Theorem and verify the theorem for $f(x) = \frac{x(x+3)}{e^{x/2}}$; $x \in [-3, 0]$

3. Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x}$

4. Find the asymptotes of the curve: $(a+x)^2(b^2+x^2) = x^2 y^2$

5. Find the pedal equation of the curve $r^2 = a^2 \cos 2\theta$

6. Evaluate $\int_0^{\pi/4} \frac{(\sin x + \cos x)}{(9+16 \sin 2x)} dx$

7. Use Beta Gamma function to evaluate $\int_0^{2a} x^5 \sqrt{2ax - x^2} dx$

8. Evaluate by using the rule of differentiation under the sign of integration.

$$\int_0^{\infty} \frac{e^{-x} \sin bx}{x} dx$$

9. Find the area of one loop of the curve $r = a \sin 3\theta$

OR

Find the volume of the solid formed by the revolution of the cardioid $r = a(1 + \cos \theta)$ about the initial line.

Find center and eccentricity of conic $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$

OR

Describe and sketch the graph of the equation $r = \frac{10}{3 + 2 \cos \theta}$

10. Find the condition that the line $lx + my + n = 0$ may be a normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

11. Show that the pair of tangents drawn from the center of a hyperbola are its asymptotes.

12. Solve the differential equation: $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$

13. Solve: $y - 2px + ayp^2 = 0$ where $p = \frac{dy}{dx}$

14. Solve the differential equation: $x \frac{dy}{dx} + y \log y = xye^x$

15. Solve the differential equation: $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - 4y = x^2$

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1. If $y = \log(x + \sqrt{a^2 + x^2})$, then show that $(a^2 + x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$ [5]

2. State and prove Lagrange's Mean Value theorem. [5]

3. Evaluate: $\lim_{x \rightarrow 0} \prod (\sin x)^{\tan x}$ [5]

4. Find the asymptote of the curve $a^2y^2 + x^2y^2 - a^2x^2 + 2ax^3 - x^4 = 0$ [5]

5. Find the radius of curvature at the origin for the curve $x^3 + y^3 = 3axy$

6. Evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$ [5]

7. Apply differentiation under integral sign to evaluate $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx$ [5]

8. Using Gamma function show that $\int_0^{\frac{\pi}{4}} \sin^4 x \cos^2 x dx = \frac{3\pi - 4}{192}$ [5]

9. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

OR

Find the volume of the solid generated by the revolution of the cardioid $r = a(1 - \cos\theta)$ about the initial line.

10. Solve: $\sin x \frac{dy}{dx} + y \cos x = x \sin x$ [5]

11. Solve: $xp^2 - 2yp + ax = 0$ where $p = \frac{dy}{dx}$ [5]

12. Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^{3x}$ [5]

13. Solve: $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x$ [5]

14. Transform the equation $x^2 - 2xy + y^2 + x - 3y = 0$ to axes through the point $(-1, 0)$ parallel to the lines bisecting the angles between the original axes. [5]

15. Find the center, length of axes and the eccentricity of the ellipse $2x^2 + 3y^2 - 4x - 12y + 13 = 0$ [5]

16. Find the length of axes and eccentricity of the conic [5]

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$$

OR

Describe and sketch the conic $r = \frac{12}{2 - 6\cos\theta}$



01 TRIBHUVAN UNIVERSITY
 INSTITUTE OF ENGINEERING
Examination Control Division
 2071 Chaitra

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- ✓ Attempt All questions.
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1. State Leibniz's theorem on Leibniz derivatives:

If $y = \sin(m \sin^{-1} x)$ then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$$

2. Assuming the validity of expansion, find the expansion of the function $\frac{e^x}{1+e^x}$ by Maclaurin's theorem.
3. Evaluate $\lim_{x \rightarrow 0} \frac{xe^x - (1+x)\log(1+x)}{x^2}$
4. Find the asymptotes of the curve $y^3 + 2xy^2 + x^2y - y + 1 = 0$
5. Find the radius of curvature of the curve $y = x^2(x-3)$ at the points where the tangent is parallel to x-axis

OR

Find the pedal equation of the curve $r^2 = a^2 \cos 2\theta$

6. Show that $\int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}} = \frac{\pi}{4}$

7. Apply differentiation under integral sign to evaluate $\int_0^{\pi/2} \frac{dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^2}$

8. Use gamma function to prove that $\int_0^1 \frac{dx}{(1-x^6)^{1/6}} = \pi/3$

9. Find the volume or surface area of solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.

10. If the line $lx+my+n=0$ is normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then show that

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$

11. Solve the locus of a point which moves in such a way that the difference of its distance from two fixed points is constant is Hyperbola.

12. Solve the differential equation $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 6x$

13. Solve $(x^2D^2 + xD + 1)y = \sin(\log x^2)$

14. Solve $y = yp^2 + 2px$ where $p = \frac{dy}{dx}$

15. Solve: $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{2x} \sin x$

16. Describe and sketch the graph of the equation $r = \frac{10}{2 - 3 \sin \theta}$

OR

Show that the conic section represented by the equation

$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$ is an ellipse. Also find its center, eccentricity, latus rectum and foci

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1. If $y = \log(x + \sqrt{a^2 + x^2})$, then show that $(a^2 + x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$ [5]

2. State and prove Logrange's Mean Value theorem. [5]

3. Evaluate: $\lim_{x \rightarrow \pi} (\sin x)^{\tan x}$ [5]

4. Find the asymptote of the curve $a^2y^2 + x^2y^2 - a^2x^2 + 2ax^3 - x^4 = 0$ [5]

5. Find the radius of curvature at the origin for the curve $x^3 + y^3 = 3axy$

6. Evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$ [5]

7. Apply differentiation under integral sign to evaluate $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx$ [5]

8. Using Gamma function show that $\int_0^{\frac{\pi}{4}} \sin^4 x \cos^2 x dx = \frac{3\pi - 4}{192}$ [5]

9. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

OR

Find the volume of the solid generated by the revolution of the cardioid $r = a(1 - \cos\theta)$ about the initial line.

10. Solve: $\sin x \frac{dy}{dx} + y \cos x = x \sin x$ [5]

11. Solve: $xp^2 - 2yp + ax = 0$ where $p = \frac{dy}{dx}$ [5]

12. Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^{3x}$ [5]

13. Solve: $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x$ [5]

14. Transform the equation $x^2 - 2xy + y^2 + x - 3y = 0$ to axes through the point $(-1, 0)$ parallel to the lines bisecting the angles between the original axes. [5]

15. Find the center, length of axes and the eccentricity of the ellipse $2x^2 + 3y^2 - 4x - 12y + 13 = 0$ [5]

16. Find the length of axes and eccentricity of the conic [5]

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$$

OR

Describe and sketch the conic $r = \frac{12}{2 - 6\cos\theta}$



2070 Chaitra

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Subject: - Engineering Mathematics I (SH401)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
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✓ 1. If $Y = \sin(m \sin^{-1}x)$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$

2. Apply Maclaurin's series to find the expansion of $\frac{e^x}{1+e^x}$ as far as the term in x^3

3. Evaluate: $\lim_{x \rightarrow a} \left(2 - \frac{x}{a}\right)^{\tan \frac{\pi x}{2a}}$

4. Find the asymptotes of the curve $x(x-y)^2 - 3(x^2 - y^2) + 8y = 0$

5. Find the pedal equation of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

6. Apply the method of differentiation under integral sign to evaluate $\int_0^{\infty} \frac{\log(1+a^2x^2)}{1+b^2x^2} dx$

7. Show that $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx = \pi \log 2$

8. Use Gamma function to prove that $\int_0^1 \frac{dx}{(1-x^6)^{\frac{1}{6}}} = \frac{\pi}{3}$

9. Find the area of two loops of the curve $a^2y^2 = a^2x^2 - x^4$

OR

Find the volume of the solid formed by the revolution of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ about the tangent at the vertex.

✓ 10. Solve the differential equation $(1+y^2) + (x - e^{\tan^{-1}y}) \frac{dy}{dx} = 0$

✓ 11. Solve: $y - 3px + ayp^2 = 0$

✓ 12. Solve: $(D^2 - 2D + 5)y = e^{2x} \cdot \sin x$

13. A resistance of 100 Ohms, an inductance of 0.5 Henry are connected in series with a battery 20 volts. Find the current in the circuit as a function of time.

14. What does the equation $3x^2 + 3y^2 + 2xy = 2$ becomes when the axes are turned through an angle 45° to the original axes.

15. Show that the locus of a point which moves in such a way that the differences of its distance from two fixed points is constant is a hyperbola.

16. Find the center, length of the axes and eccentricity of the conic $2x^2 + 3y^2 - 4x - 12y + 13 = 0$

OR

Describe and sketch the graph of the polar equation of conic $r = \frac{10 \operatorname{cosec}\theta}{2 \operatorname{cosec}\theta + 3}$

Handwritten notes and diagrams at the bottom left, including sketches of conic sections and calculations.

Handwritten notes and diagrams at the bottom right, including a sketch of a hyperbola and calculations.

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1. If $y = \log(x + \sqrt{a^2 + x^2})$ show that $(a^2 + x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$
2. State and prove Lagrange's Mean Value theorem.
3. If $\lim_{x \rightarrow 0} \frac{a \sin x - \sin 2x}{\tan^3 x}$ is finite, find the value of a and the limit.
4. Find asymptotes of $(x^2 - y^2)^2 - 2(x^2 + y^2) + x - 1 = 0$
5. Find the radius of curvature at any point (x, y) for the curve $x^{2/3} + y^{2/3} = a^{2/3}$
6. Prove that $\int_0^\infty \frac{\sin bx}{x} dx = \frac{\pi}{2} (b > 0)$
7. Use Beta and Gamma function to evaluate $\int_0^{2a} x^5 \sqrt{2ax - x^2} dx$
8. Evaluate $\int_0^\infty \frac{e^{-x} \sin bx}{x} dx$ by using the rule of differentiation under the sign of integration.
9. Find the volume of the solid formed by the revolution of the cardioid $r = a(1 + \cos\theta)$ about initial line.

OR

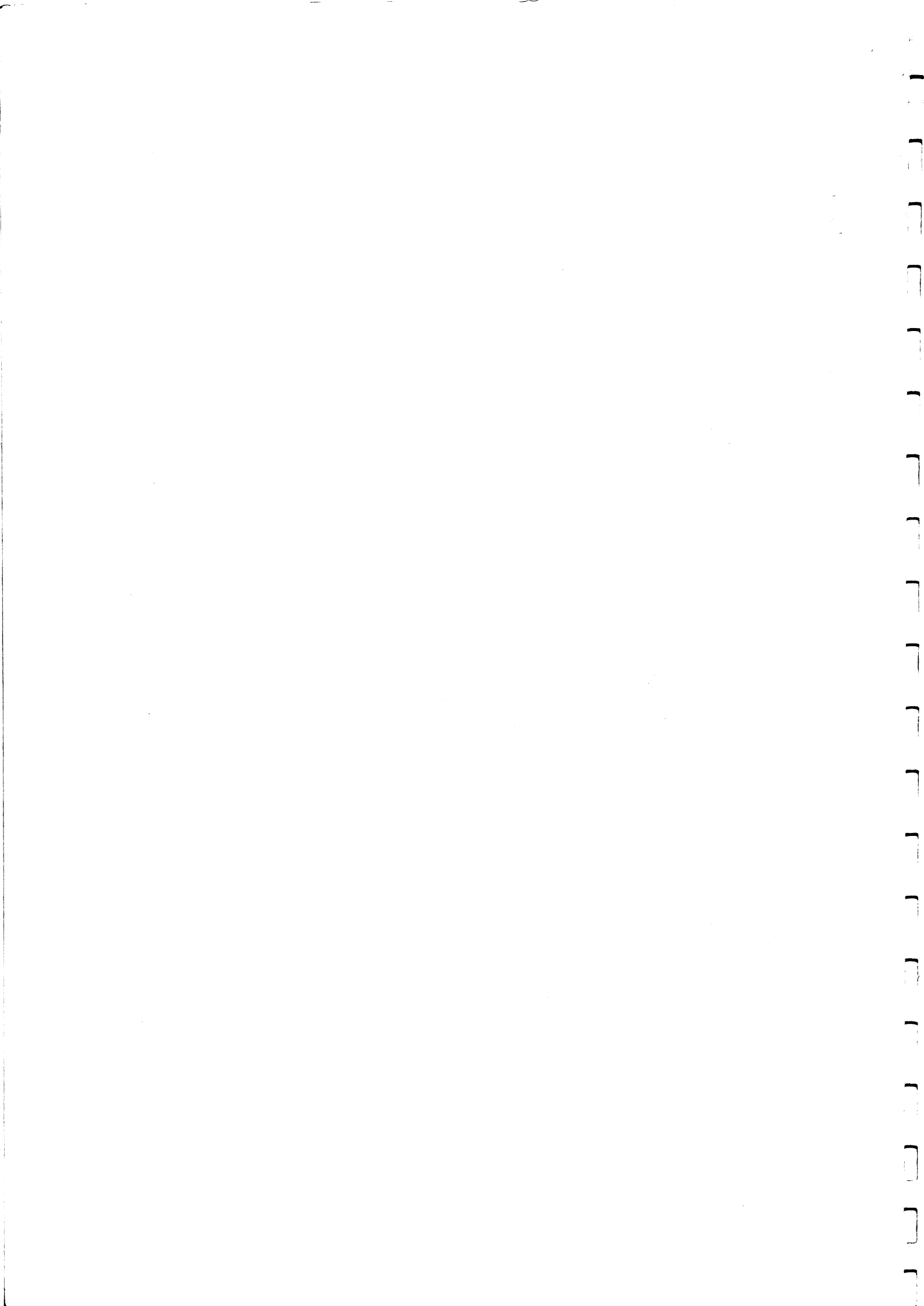
Find the area bounded by the curve $x^2y = a^2(a - y)$ and the x-axes.

10. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$
11. Solve the differential equation $x \frac{dy}{dx} + y \log y = xye^x$
12. Solve the differential equation $\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = e^x + e^{-x}$
13. Solve $y = px - \sqrt{m^2 + p^2}$ where $p = \frac{dy}{dx}$

OR

A resistance of 100 ohms, an inductance of 0.5 henry are connected in series with a battery of 20 volts. Find the current in the circuit as a function of time.

14. Solve that locus of a point which moves in such a way that the differences of its distance from two fixed points is constant is Hyperbola.
15. Find the equation of ellipse of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b$
16. Describe and sketch the graph of the equation $r = \frac{4 \sec \theta}{2 \sec \theta - 1}$



Exam.	New Back (2066 Batch & Later)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ All questions carry equal marks.
- ✓ Assume suitable data if necessary.

1. If $y = \log(x + \sqrt{a^2 + x^2})$, show that $(a^2 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0$.

2. State and prove Lagrange's mean value theorem.

3. Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x}$.

4. Find the asymptotes of the curve $(x^2 - y^2)(x + 2y + 1) + x + y + 1 = 0$.

5. Show that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the radius of curvature at the extremity of the major axis is equal to half of the latus rectum.

6. Evaluate: $\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$.

7. Use Gamma function to prove that $\int_0^1 \frac{dx}{(1-x^6)^{1/6}} = \frac{\pi}{3}$.

8. Using method of differentiation under integral sign, evaluate: $\int_0^{\infty} \frac{e^{-x} \sin bx}{x} dx$.

9. Find the area bounded by the cardioid, $r = a(1 + \cos\theta)$.

OR

Find the volume of the solid formed by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.

10. Find the angle through which the axes must be turned so that the equation $ax^2 + 2hxy + by^2 = 0$ may become an equation having no term involving xy .

11. Obtain the equation of an ellipse in the standard form.

12. Find the centre of the conic $3x^2 + 8xy - 3y^2 - 40x - 20y + 50 = 0$.

13. Solve the differential equation $(x + y + 1) \frac{dy}{dx} = 1$.

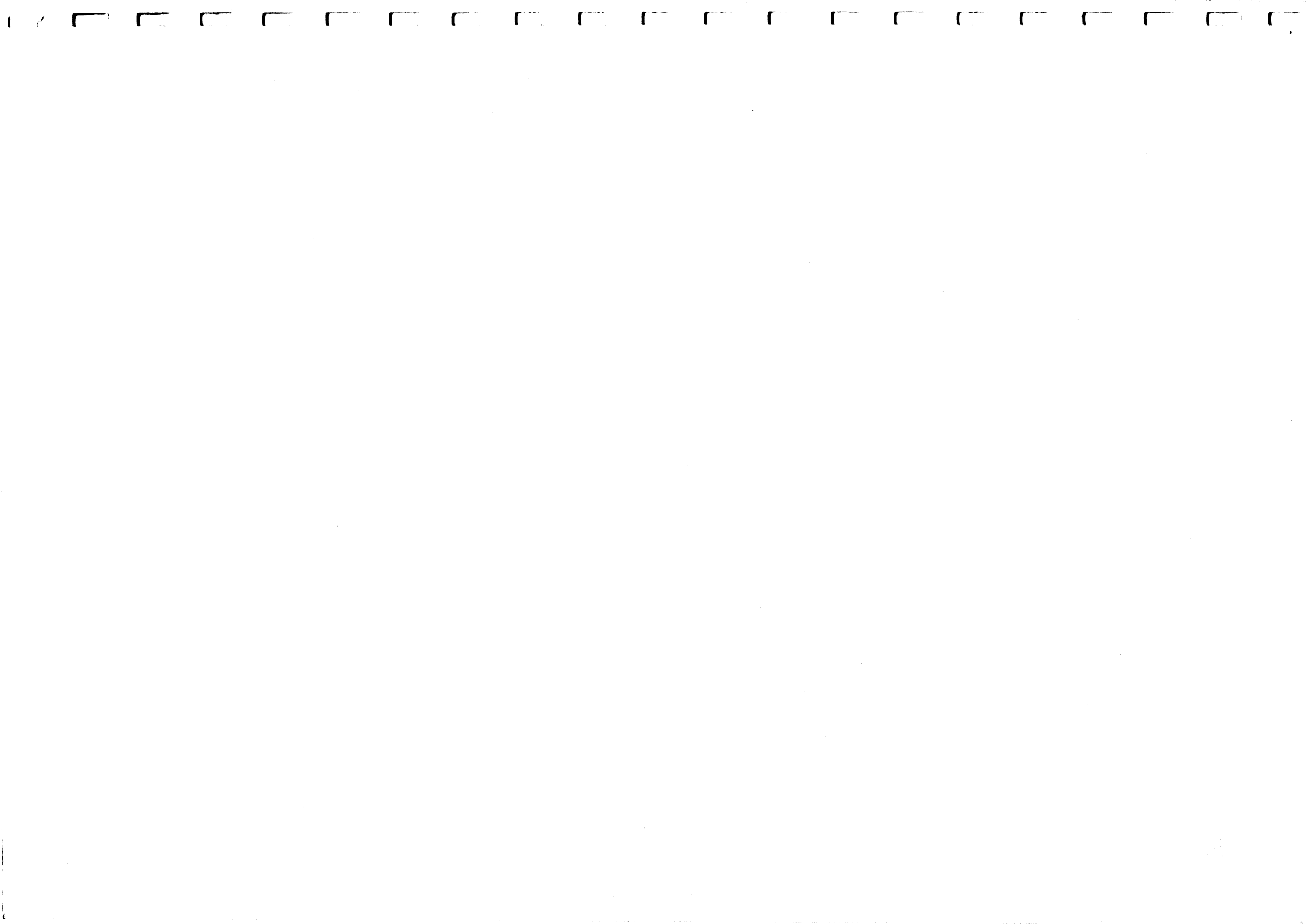
14. Find the general solution of the differential equation: $p^3 - 4xyp + 3y^2 = 0$.

15. Find the general solution of the differential equation: $(D^2 + 2D + 1)y = e^x \cos x$.

16. Newton's law of cooling states that "The temperature of an object changes at a rate proportional to the difference of temperatures between the object and its surroundings". Supposing water at a temperature 100°C cools to 80°C in 10 minutes, in a room maintained at 30°C , find when the temperature of water will become 40°C .

OR

Solve: $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = x$



Exam. Level	Regular / Back		
	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ All questions carry equal marks.
- ✓ Assume suitable data if necessary.

1. If $y = a \cos(\log x) + b \sin(\log x)$. Prove that $x^2 \cdot y_{n+2} + (2n+1)x \cdot y_{n+1} + (n^2 + 1)y_n = 0$.
2. State and prove Rolle's theorem.

3. Determine the values of a, b, c, so that $\lim_{x \rightarrow 0} \frac{(a + b \cos x)x - c \cdot \sin x}{x^5} = 1$.

4. Find the asymptotes of the curve $(x + y)^2(x + 2y + 2) = x + 9y - 2$.

5. If e_1 and e_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2 = 4ax$, prove that $e_1^{-2/3} + e_2^{-2/3} = (2a)^{-2/3}$.

6. Prove that $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx = \frac{\pi^2}{4}$.

7. Apply the method of differentiation under integral sign to prove:

$$\int_0^{\pi/2} \frac{dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^2} = \frac{\pi(a^2 + b^2)}{4a^3b^3}$$

8. Use Gamma function to prove that $\int_0^1 \frac{dx}{(1-x^6)^{1/6}} = \frac{\pi}{3}$.

9. Find the area bounded by the curve $x^2y = a^2(a-y)$ and the x axis.

OR

Find the volume of the solid formed by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.

10. Solve the differential equation: $(1 + y^2) + (x - e^{\tan^{-1}y}) \frac{dy}{dx} = 0$.

11. Solve: $xy^2(p^2 + 2) = 2py^3 + x^3$

12. solve: $(D^2 - 2D + 5)y = e^{2x} \cdot \sin x$

13. Solve the differential equation: $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$

14. What does the equation $3x^2 + 3y^2 + 2xy = 2$ becomes when the axes are turned through an angle 45° to the original axis.

OR

Describe and Sketch the graph of the conic $r = \frac{10 \cos e\theta}{2 \cos e\theta + 3}$.

15. Derive the equation of Ellipse in the standard form.

16. Find the equation of tangents to the hyperbola $3x^2 - 4y^2 = 12$ which are perpendicular to the line $x - y + 2 = 0$. Also find the point of contact.

Exam.	Regular/Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. If $y = e^{a \tan^{-1} x}$, prove that $(1 + x^2)y_{n+2} + (2nx + 2x - a)y_{n+1} + n(n+1)y_n = 0$. 5

2. State and prove Lagrange's mean value theorem.

3. Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x}}$

4. Find the asymptotes of the curve $(x + y)^2(x + 2y + z) = x + 9y - 2$.

5. Find the radius of curvature of the curve $r = a(1 - \cos\theta)$.

6. Apply the method of differentiation under integral sign to evaluate $\int_0^{\infty} \frac{\tan^{-1}(ax)}{x(1+x^2)} dx$.

7. Prove that $\int_0^{\pi/2} \frac{\sin^2 x dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$.

8. Use Gamma function to prove $\int_0^{\pi/6} \cos^4 3\theta \sin^2 6\theta = \frac{5\pi}{192}$. 5

9. Find, by method of integration, the area of the loop of the curve $ay^2 = x^2(a - x)$.

10. Solve the differential equation $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$. 5

11. Solve $y = yp^2 + 2px$, where $p = dy/dx$. 5

12. Solve $(D^2 - 3D + 2)y = x^2 + x$. 5

13. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference of temperature between the object and its surroundings. Supposing water at 100°C cools to 80°C in 10 minutes, in a room temperature of 30°C , find when the temperature of water will become 40°C ?

OR

Solve the differential equation $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \log x$.

14. Find the condition that the line $lx + my + n = 0$ may be the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 5

15. Derive the equation of a hyperbola in standard form. 5

16. Find the centre, length of axes and eccentricity of the conic $2x^2 + 3y^2 - 4x - 12y + 13 = 0$.

OR

Identify and sketch the conic $r = \frac{10}{3 + 2 \cos \theta}$.

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Mathematics I

- ✓ Candidates are required to give their answers in their own words as far as practicable.
 ✓ Attempt All questions.
 ✓ The figures in the margin indicate Full Marks.
 ✓ Assume suitable data if necessary.
- ✓ Find the angle of intersection of the pair of curves $r^n = a^n \cos n\theta$ and $r^n = a^n \sin n\theta$. [5]
- OR
- If $y = a \cos(\log x) + b \sin(\log x)$. Prove that $x^2 y_{n+2} + (2n+1)x y_{n+1} + (x^2 + 1)y_n = 0$
- ✓ State Rolle's theorem and verify it for the function $f(x) = x(x+3)e^{-(x/2)}$, $x \in [-3, 0]$ [5]
3. Evaluate: $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x}$ [1+4]
- ✓ A cone is circumscribed to a sphere of radius r . Show that when the volume of the cone is least its altitude is $4r$ and its semivertical angle is $\sin^{-1}(1/3)$. [5]
- ✓ Find the asymptotes of the curve $(x+y)^2(x+2y+2) = x+9y-2$. [5]
- OR
- Find the radius of curvature at any point (x, y) for the curve $x^{2/3} + y^{2/3} = a^{2/3}$.
6. Integrate any three [10]
- a) $\int \frac{x.e^x}{(1+x)^2} .dx$ b) $\int_0^1 \frac{\log(1+x)}{1+x^2} .dx$
- c) $\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}} .dx$ d) $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{1+\sqrt{\cot x}} .dx$
7. Evaluate $\int_1^4 x^3 dx$ by the method of summation. [5]
8. Obtain reduction formula for $\int \cot^n x dx$ and hence integrate $\int \cot^7 x dx$. [3]
- OR
- Using Gamma function show that $\int_0^{\infty} e^{-x^4} .x^2 dx \times \int_0^{\infty} e^{-x^4} .dx = \frac{\pi}{8\sqrt{2}}$
9. Find the area bounded by the cardioid $r = a(1 + \cos\theta)$ [5]
- OR
- Find the volume of the solid formed by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.
10. Solve any three of the following differential equations. [15]
- a) $x dy - y dx = \sqrt{x^2 + y^2} .dx$ b) $x \frac{dy}{dx} + y \log y = xy.e^x$
- c) $y - 2px + ap^2.y = 0$ d) $(D^2 - 3D + 2)y = e^x$
11. If the axes be turned through an angle $\tan\theta = 2$. What does the equation $4xy - 3x^2 = a^2$ becomes? [5]
12. Find the equation of an ellipse in the standard form. [5]
13. If e_1 and e_2 are the eccentricities of the hyperbola, and it conjugate respectively. Then prove that $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$. [5]

Exam.	Regular/Back		
	Level	BE	Full Marks
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Mathematics I

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Find the angle between the curves $r = a \sin 2\theta$, $r = a \cos 2\theta$. [5]

OR

If $y = (x^2 - 1)^n$, prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

2. State and prove Lagrange's mean value theorem. [5]

3. Evaluate: $\lim_{x \rightarrow 0} (\cot x)^{\frac{1}{\log x}}$ [5]

4. Find the surface of the right circular cylinder of greatest surface which can be inscribed in a sphere of radius r . [5]

5. Find the asymptotes of the curve $(x^2 - y^2)(x + 2y + 1) + x + y + 1 = 0$. [5]

OR

Show that the radius of curvature for the curve $r^m = a^m \cos m\theta$ is $\frac{a^m}{(m+1)r^{m-1}}$.

6. Integrate any three: [10]

a) $\int \frac{\cos x dx}{(1 + \sin x)(2 + \sin x)}$

b) $\int_0^{\pi/4} \frac{\sin 2\theta d\theta}{\sin^4 \theta + \cos^4 \theta}$

c) $\int_0^{\pi/2} \frac{\sqrt{\cot x} dx}{1 + \sqrt{\cot x}}$

d) $\int_{-1}^2 \frac{dx}{x^3}$

7. Evaluate $\int_0^1 \sqrt{x} dx$ by the method of summation. [5]

8. Obtain a reduction formula for $\int \sec^n x dx$ and hence find $\int \sec^6 x dx$. [5]

OR

Evaluate $\int_0^1 \frac{dx}{(1 - x^6)^{1/6}}$

9. Find the area of a loop of the curve $a^2 y^2 = a^2 x^2 - x^4$. [5]

OR

Find the volume of the solid generated by revolving the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the axis of x .

10. Solve any three of the following differential equations. [15]

a) $(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0$

b) $\cos x dy = y(\sin x - y)dx$

c) $p^2 - py + x = 0$; where $p = \frac{dy}{dx}$

d) $(D^2 - 3D + 2)y = x^2 + x$

11. Find the changed form of the equation $3x^2 + 3y^2 + 2xy = 2$ when the axes are turned through 45° the origin remaining fixed. [5]

12. The line $x + y = 0$ is a directrix of an ellipse, the point $(2,2)$ is the corresponding focus. If the eccentricity be $1/3$, find the equation of the other directrix. [5]

13. Find the equation of the hyperbola in the standard form [5]